10 ATMOSPHERIC FORCES & WINDS

Contents

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Winds power our wind turbines, push our sailboats, cool our houses, and dry our laundry. But winds can also be destructive — in hurricanes, thunderstorms, or mountain downslope windstorms. We design our bridges and skyscrapers to withstand wind gusts. Airplane flights are planned to compensate for headwinds and crosswinds.

Winds are driven by forces acting on air. But these forces can be altered by heat and moisture carried by the air, resulting in a complex interplay we call weather. Newton's laws of motion describe how forces cause winds — a topic called **dynamics**.

Many forces such as pressure-gradient, advection, and frictional drag can act in all directions. Inertia creates an apparent centrifugal force, caused when centripetal force (an imbalance of other forces) makes wind change direction. Local gravity acts mostly in the vertical. But a local horizontal component of gravity due to Earth's non-spherical shape, combined with the contribution to centrifugal force due to Earth's rotation, results in a net force called Coriolis force.

These different forces are present in different amounts at different places and times, causing large variability in the winds. For example, Fig. 10.1 shows changing wind speed and direction around a low-pressure center. In this chapter we explore forces, winds, and the dynamics that link them.

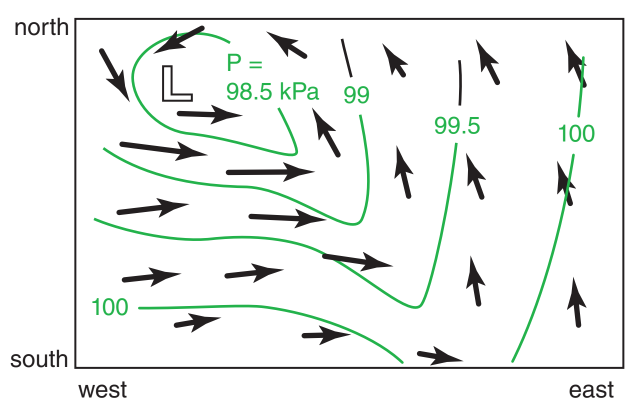


Figure 10.1
Winds (arrows) around a low-pressure center (L) in the N. Hemisphere. Green lines are isobars of sea-level pressure (P).

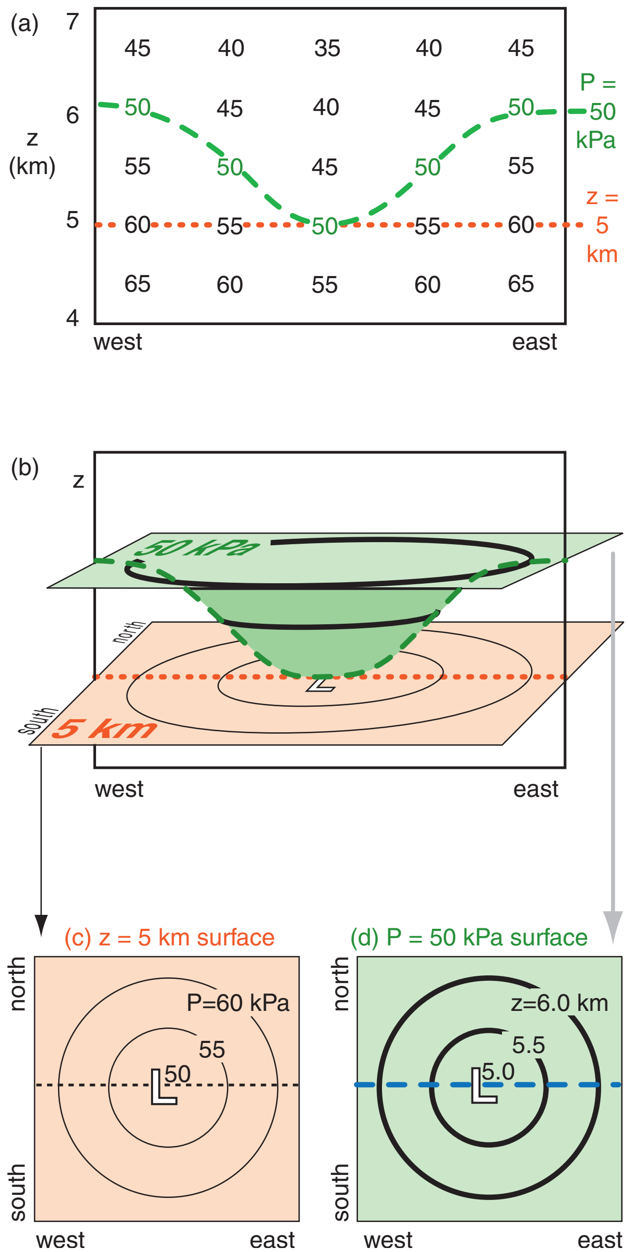


Figure 10.2

Sketch of the similarity of (c) pressures drawn on a constant height surface to (d) heights drawn on a constant pressure surface. At any one height such as z = 5 km (shown by the thin dotted line in the vertical cross section of Fig. a), pressures at one location on the map might differ from pressure at other locations. In this example, a pressure of 50 kPa is located midway between the east and west limits of the domain at 5 km altitude. Pressure always decreases with increasing height z, as sketched in the vertical cross section of (a). Thus, at the other locations on the cross section, the 50 kPa pressure will be found at higher altitudes, as sketched by the thick dashed line. This thick dashed line and the corresponding thin dotted straight line are copied into the 3-D view of the same scenario is sketched in Fig. b. "L" indicates the cyclone center, having low pressure and low heights.

10.1. WINDS AND WEATHER MAPS

10.1.1. Heights on Constant-Pressure Surfaces

Pressure-gradient force is the most important force because it is the only one that can drive winds in the horizontal. Other horizontal forces can alter an existing wind, but cannot create a wind from calm air. All the forces, including pressure-gradient force, are explained in the next sections. However, to understand the pressure gradient, we must first understand pressure and its atmospheric variation.

We can create weather maps showing values of the pressures measured at different horizontal locations all at the same altitude, such as at mean-sea-level (MSL). Such a map is called a **constant-height map**. However, one of the peculiarities of meteorology is that we can also create maps on other surfaces, such as on a surface connecting points of equal pressure. This is called an **isobaric map**. Both types of maps are used extensively in meteorology, so you should learn how they are related.

In Cartesian coordinates (x, y, z), z is height above some reference level, such as the ground or sea level. Sometimes we use geopotential height H in place of z, giving a coordinate set of (x, y, H) (see Chapter 1).

Can we use pressure as an alternative vertical coordinate instead of z? The answer is yes, because pressure changes monotonically with altitude. The word **monotonic** means that the value of the dependent variable changes in only one direction (never decreases, or never increases) as the value of the independent variable increases. Because P never increases with increasing z, it is indeed monotonic, allowing us to define **pressure coordinates** (x, y, P).

An isobaric surface is a conceptual curved surface that connects points of equal pressure, such as the shaded surface in Fig. 10.2b. The surface is higher above sea level in high-pressure regions, and lower in low-pressure regions. Hence the height contour lines for an isobaric surface are good surrogates for pressures lines (isobars) on a constant height map. Contours on an isobaric map are analogous to elevation contours on a topographic map; namely, the map itself is flat, but the contours indicate the height of the actual surface above sea level.

High pressures on a constant height map correspond to high heights of an isobaric map. Similarly, regions on a constant-height map that have tight **packing** (close spacing) of isobars correspond to regions on isobaric maps that have tight packing of height contours, both of which are regions of strong pressure gradients that can drive strong winds. This one-to-one correspondence of both types of maps (Figs. 10.2c & d) makes it easier for you to use them interchangeably.

Isobaric surfaces can intersect the ground, but two different isobaric surfaces can never intersect because it is impossible to have two different pressures at the same point. Due to the smooth monotonic decrease of pressure with height, isobaric surfaces cannot have folds or creases.

We will use isobaric charts for most of the upper-air weather maps in this book when describing upper-air features (mostly for historical reasons; see INFO box). Fig. 10.3 is a sample weather map showing height contours of the 50 kPa isobaric surface.

10.1.2. Plotting Winds

Symbols on weather maps are like musical notes in a score — they are a shorthand notation that concisely expresses information. For winds, the symbol is an arrow with feathers (or barbs and pennants). The tip of the arrow is plotted over the observation (weather-station) location, and the arrow shaft is aligned so that the arrow points toward where the wind is going. The number and size of the feathers indicates the wind speed (Table 10-1, copied from Table 9-9). Fig. 10.3 illustrates wind barbs.

Table 10-1. Interpretation of wind barbs.

Symbol	Wind Speed	Description
	calm	two concentric circles
	1 - 2 speed units	shaft with no barbs
	5 speed units	a half barb (half line)
	10 speed units	each full barb (full line)
	50 speed units	each pennant (triangle)

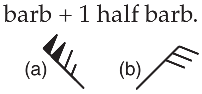
• The total speed is the sum of all barbs and pennants. For example,  indicates a wind from the west at speed 75 units. Arrow tip is at the observation location.
• CAUTION: Different organizations use different speed units, such as knots, m s^{-1}, miles h^{-1}, km h^{-1}, etc. Look for a legend to explain the units. When in doubt, assume knots — the WMO standard. For unit conversion, a good approximation is 1 m s^{-1} ≈ 2 knots.

Sample Application

Draw wind barb symbol for winds from the:
(a) northwest at 115 knots; (b) northeast at 30 knots.

Find the Answer

(a) 115 knots = 2 pennants + 1 full barb + 1 half barb.
(b) 30 knots = 3 full barbs



Check: Consistent with Table 10-1.
Exposition: Feathers (barbs & pennants) should be on the side of the shaft that would be towards low pressure if the wind were geostrophic.

INFO • Why use isobaric maps?

There are five reasons for using isobaric charts.

1) During the last century, the **radiosonde** (a weather sensor hanging from a free helium balloon that rises into the upper troposphere and lower stratosphere) could not measure its geometric height, so instead it reported temperature and humidity as a function of pressure. For this reason, **upper-air charts** (i.e., maps showing weather above the ground) traditionally have been drawn on isobaric maps.

2) Aircraft altimeters are really pressure gauges. Aircraft assigned by air-traffic control to a specific "altitude" above 18,000 feet MSL will actually fly along an isobaric surface. Many weather observations and forecasts are motivated by aviation needs.

3) Air pressure is created by the weight of air molecules. Thus, every point on an isobaric map has the same mass of air molecules above it.

4) An advantage of using equations of motion in pressure coordinates is that you do not need to consider density, which is not routinely observed.

5) Some numerical weather prediction models use reference pressures that vary hydrostatically with altitude.

Items (1) and (5) are less important these days, because modern radiosondes use **GPS** (**Global Positioning System**) to determine their (x, y, z) position. So they report all meteorological variables (including pressure) as a function of z. Also, some of the modern weather forecast models do not use pressure as the vertical coordinate. Perhaps future weather analyses and numerical predictions will be shown on constant-height maps.

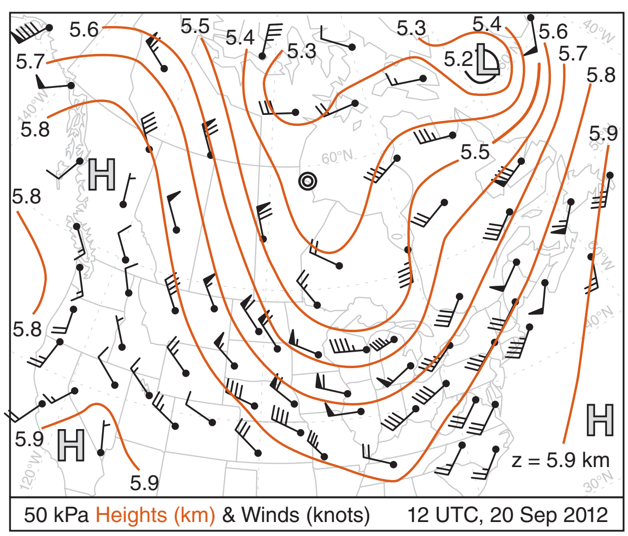


Figure 10.3
Winds (1 knot ≈ 0.5 m s^{-1}) and heights (km) on the 50 kPa isobaric surface. The relative maxima and minima are labeled as **H** *(high heights) and* **L** *(low heights). Table 10-1 explains winds.*

INFO • Newton's Laws of Motion

Isaac Newton's published his laws in Latin, the language of **natural philosophy** (science) at the time (1687). Here is the translation from Newton's *Philosophiæ Naturalis Principia Mathematica* ("Mathematical Principles of Natural Philosophy"):

"**Law I**. Every body perseveres in its state of being at rest or of moving uniformly straight forward, except inasmuch as it is compelled by impressed forces to change its state.

"**Law II**. Change in motion is proportional to the motive force impressed and takes place following the straight line along which that force is impressed.

"**Law III**. To any action, there is always a contrary, equal reaction; in other words, the actions of two bodies each upon the other are always equal and opposite in direction.

"**Corollary 1**. A body under the joint action of forces traverses the diagonal of a parallelogram in the same time as it describes the sides under their separate actions."

Sample Application

If a 1200 kg car accelerates from 0 to 100 km h^{-1} in 7 s, heading north, then: (a) What is its average acceleration? (b) What vector force caused this acceleration?

Find the Answer

Given: $\vec{V}_{initial} = 0$, $\vec{V}_{final} = 100$ km h^{-1} = 27.8 m s^{-1}
$t_{initial} = 0$, $t_{final} = 7$ s. Direction is north.
$m = 1200$ kg.

Find: (a) $\vec{a}$ = ? m·s^{-2} , (b) $\vec{F}$ = ? N

(a) Apply eq. (10.2): $\vec{a} = \dfrac{\Delta \vec{V}}{\Delta t}$

= (27.8 – 0 m s^{-1}) / (7 – 0 s) = __3.97__ m·s^{-2} __to the north__

(b) Apply eq. (10.1): $\vec{F}$ = (1200 kg) · (3.97 m·s^{-2})
= __4766 N to the north__
where 1 N = 1 kg·m·s^{-2} (see Appendix A).

Check: Physics and units are reasonable.
Exposition: My small car can accelerate from 0 to 100 km in 20 seconds, if I am lucky. Greater acceleration consumes more fuel, so to save fuel and money, you should accelerate more slowly when you drive.

10.2. NEWTON'S 2ND LAW

10.2.1. Lagrangian

For a Lagrangian framework (where the coordinate system follows the moving object), **Newton's Second Law of Motion** is

$$\vec{F} = m \cdot \vec{a} \qquad \bullet(10.1)$$

where $\vec{F}$ is a force vector, m is mass of the object, and $\vec{a}$ is the acceleration vector of the object. Namely, the object accelerates in the direction of the applied force.

Acceleration is the velocity $\vec{V}$ change during a short time interval Δt:

$$\vec{a} = \frac{\Delta \vec{V}}{\Delta t} \qquad (10.2)$$

Plugging eq. (10.2) into (10.1) gives:

$$\vec{F} = m \cdot \frac{\Delta \vec{V}}{\Delta t} \qquad (10.3a)$$

Recall that **momentum** is defined as $m \cdot \vec{V}$. Thus, if the object's mass is constant, you can rewrite Newton's 2nd Law as **Lagrangian momentum budget**:

$$\vec{F} = \frac{\Delta(m \cdot \vec{V})}{\Delta t} \qquad (10.3b)$$

Namely, this equation allows you to forecast the rate of change of the object's momentum.

If the object is a collection of air molecules moving together as an **air parcel**, then eq. (10.3a) allows you to forecast the movement of the air (i.e., the **wind**). Often many forces act simultaneously on an air parcel, so we should rewrite eq. (10.3a) in terms of the net force:

$$\frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{F}_{net}}{m} \qquad (10.4)$$

where $\vec{F}_{net}$ is the vector sum of all applied forces, as given by Newton's Corollary 1 (see the INFO box).

For situations where $\vec{F}_{net}/m = 0$, eq. (10.4) tells us that the flow will maintain constant velocity due to **inertia**. Namely, $\Delta \vec{V}/\Delta t = 0$ implies that $\vec{V}$ = constant (not necessarily that $\vec{V} = 0$).

In Chapter 1 we defined the (U, V, W) wind components in the (x, y, z) coordinate directions (positive toward the East, North, and up). Thus, we can split eq. (10.4) into separate **scalar** (i.e., non-vector) equations for each wind component:

$$\frac{\Delta U}{\Delta t} = \frac{F_{x\,net}}{m} \qquad \bullet(10.5a)$$

$$\frac{\Delta V}{\Delta t} = \frac{F_{y\,net}}{m} \qquad \bullet(10.5b)$$

$$\frac{\Delta W}{\Delta t} = \frac{F_{z\,net}}{m} \qquad \bullet(10.5c)$$

where $F_{x\,net}$ is the sum of the x-component of all the applied forces, and similar for $F_{y\,net}$ and $F_{z\,net}$.

From the definition of Δ = final – initial, you can expand $\Delta U/\Delta t$ to be $[U(t+\Delta t) - U(t)]/\Delta t$. With similar expansions for $\Delta V/\Delta t$ and $\Delta W/\Delta t$, eq. (10.5) becomes

$$U(t+\Delta t) = U(t) + \frac{F_{x\,net}}{m}\cdot\Delta t \qquad \bullet(10.6a)$$

$$V(t+\Delta t) = V(t) + \frac{F_{y\,net}}{m}\cdot\Delta t \qquad \bullet(10.6b)$$

$$W(t+\Delta t) = W(t) + \frac{F_{z\,net}}{m}\cdot\Delta t \qquad \bullet(10.6c)$$

These are forecast equations for the wind, and are known as the **equations of motion**. The Numerical Weather Prediction (NWP) chapter shows how the equations of motion are combined with budget equations for heat, moisture, and mass to forecast the weather.

10.2.2. Eulerian

While Newton's 2nd Law defines the fundamental dynamics, we cannot use it very easily because it requires a coordinate system that moves with the air. Instead, we want to apply it to a fixed location (i.e., an **Eulerian** framework), such as over your house. The only change needed is to include a new term called **advection** along with the other forces, when computing the net force F_{net} in each direction. All these forces are explained in the next section.

But knowing the forces, we need additional information to use eqs. (10.6) — we need the initial winds [$U(t)$, $V(t)$, $W(t)$] to use for the first terms on the right side of eqs. (10.6). Hence, to make numerical weather forecasts, we must first observe the current weather and create an **analysis** of it. This corresponds to an **initial-value problem** in mathematics.

Average horizontal winds are often 100 times stronger than vertical winds, except in thunderstorms and near mountains. We will focus on horizontal forces and winds first.

Sample Application

Initially still air is acted on by force $F_{y\,net}/m = 5\text{x}10^{-4}$ m·s^{-2} . Find the final wind speed after 30 minutes.

Find the Answer

Given: $V(0) = 0$, $F_{y\,net}/m = 5\text{x}10^{-4}$ m·s^{-2} , $\Delta t = 1800$ s
Find: $V(\Delta t) = ?$ m s^{-1}. Assume: $U = W = 0$.

Apply eq. (10.6b): $V(t+\Delta t) = V(t) + \Delta t \cdot (F_{y\,net}/m)$
$= 0 + (1800\text{s})\cdot(5\text{x}10^{-4}\ \text{m·s}^{-2}) = \underline{\mathbf{0.9\ m\ s^{-1}}}$.

Check: Physics and units are reasonable.
Exposition: This wind toward the north (i.e., from 180°) is slow. But continued forcing over more time could make it faster.

A SCIENTIFIC PERSPECTIVE • Creativity

As a child at Woolsthorpe, his mother's farm in England, Isaac Newton built clocks, sundials, and model windmills. He was an average student, but his schoolmaster thought Isaac had potential, and recommended that he attend university.

Isaac started Cambridge University in 1661. He was 18 years old, and needed to work at odd jobs to pay for his schooling. Just before the plague hit in 1665, he graduated with a B.A. But the plague was spreading quickly, and within 3 months had killed 10% of London residents. So Cambridge University was closed for 18 months, and all the students were sent home.

While isolated at his mother's farm, he continued his scientific studies independently. This included much of the foundation work on the laws of motion, including the co-invention of calculus and the explanation of gravitational force. To test his laws of motion, he built his own telescope to study the motion of planets. But while trying to improve his telescope, he made significant advances in optics, and invented the reflecting telescope. He was 23 - 24 years old.

It is often the young women and men who are most creative — in the sciences as well as the arts. Enhancing this creativity is the fact that these young people have not yet been overly swayed (perhaps misguided) in their thinking by the works of others. Thus, they are free to experiment and make their own mistakes and discoveries.

You have an opportunity to be creative. Be wary of building on the works of others, because subconsciously you will be steered in their same direction of thought. Instead, I encourage you to be brave, and explore novel, radical ideas.

This recommendation may seem paradoxical. You are reading my book describing the meteorological advances of others, yet I discourage you from reading about such advances. You must decide on the best balance for you.

10.3. HORIZONTAL FORCES

Five forces contribute to net horizontal accelerations that control horizontal winds: **pressure-gradient force** (PG), **advection** (AD), **centrifugal force** (CN), **Coriolis force** (CF), and **turbulent drag** (TD):

$$\frac{F_{x\,net}}{m} = \frac{F_{x\,AD}}{m} + \frac{F_{x\,PG}}{m} + \frac{F_{x\,CN}}{m} + \frac{F_{x\,CF}}{m} + \frac{F_{x\,TD}}{m} \tag{10.7a}$$

$$\frac{F_{y\,net}}{m} = \frac{F_{y\,AD}}{m} + \frac{F_{y\,PG}}{m} + \frac{F_{y\,CN}}{m} + \frac{F_{y\,CF}}{m} + \frac{F_{y\,TD}}{m} \tag{10.7b}$$

Centrifugal force is an apparent force that allows us to include inertial effects for winds that move in a curved line. Coriolis force, explained in detail later, includes the gravitational and compound centrifugal forces on a non-spherical Earth. In the equations above, force per unit mass has units of N kg^{-1}. These units are equivalent to units of acceleration (m·s^{-2}, see Appendix A), which we will use here.

10.3.1. Advection of Horizontal Momentum

Advection is not a true force. Yet it can cause a change of wind speed at a fixed location in Eulerian coordinates, so we will treat it like a force here. The wind moving past a point can carry **specific momentum** (i.e., momentum per unit mass). Recall that momentum is defined as mass times velocity, hence specific momentum equals the velocity (i.e., the wind) by definition. Thus, the wind can move (**advect**) different winds to your fixed location.

This is illustrated in Fig. 10.4a. Consider a mass of air (grey box) with slow U wind (5 m s^{-1}) in the north and faster U wind (10 m s^{-1}) in the south. Thus, U decreases toward the north, giving $\Delta U/\Delta y$ = *negative*. This whole air mass is advected toward the north over a fixed weather station "O" by a south wind (V = *positive*). At the later time sketched in Fig. 10.4b, a west wind of 5 m s^{-1} is measured at "O". Even later, at the time of Fig. 10.4c, the west wind has increased to 10 m s^{-1} at the weather station. The rate of increase of U at "O" is larger for faster advection (V), and when $\Delta U/\Delta y$ is more negative.

Thus, $\Delta U/\Delta t = -V \cdot \Delta U/\Delta y$ for this example. The advection term on the RHS causes an acceleration of U wind on the LHS, and thus acts like a force per unit mass: $\Delta U/\Delta t = F_{x\,AD}/m = -V \cdot \Delta U/\Delta y$.

You must always include advection when momentum-budget equations are written in Eulerian frameworks. This is similar to the advection terms in the moisture- and heat-budget Eulerian equations that were in earlier chapters.

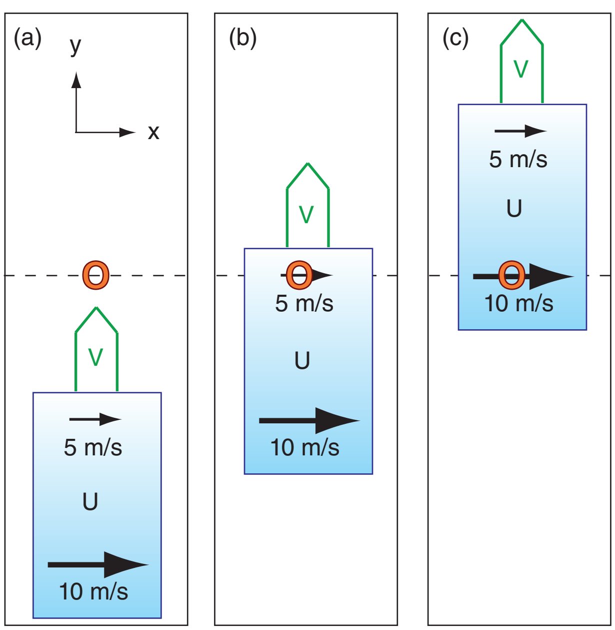


Figure 10.4
Illustration of V advection of U wind. "O" is a fixed weather station. Grey box is an air mass containing a gradient of U wind. Initial state (a) and later states (b and c).

For advection, the horizontal force components are

$$\frac{F_{x\,AD}}{m} = -U\cdot\frac{\Delta U}{\Delta x} - V\cdot\frac{\Delta U}{\Delta y} - W\cdot\frac{\Delta U}{\Delta z} \qquad \bullet(10.8a)$$

$$\frac{F_{y\,AD}}{m} = -U\cdot\frac{\Delta V}{\Delta x} - V\cdot\frac{\Delta V}{\Delta y} - W\cdot\frac{\Delta V}{\Delta z} \qquad \bullet(10.8b)$$

Recall that a **gradient** is defined as change across a distance, such as $\Delta V/\Delta y$. With no gradient, the wind cannot cause accelerations.

Vertical advection of horizontal wind ($-W\cdot\Delta U/\Delta z$ in eq. 10.8a, and $-W\cdot\Delta V/\Delta z$ in eq. 10.8b) is often very small outside of thunderstorms.

10.3.2. Horizontal Pressure-Gradient Force

In regions where the pressure changes with distance (i.e., a **pressure gradient**), there is a force from high to low pressure. On weather maps, this force is at right angles to the height contours or isobars, directly from high heights or high pressures to low. Greater gradients (shown by a tighter packing of isobars; i.e., smaller spacing Δd between isobars on weather maps) cause greater pressure-gradient force (Fig. 10.5). Pressure-gradient force is independent of wind speed, and thus can act on winds of any speed (including calm) and direction.

For pressure-gradient force, the horizontal components are:

$$\frac{F_{x\,PG}}{m} = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta x} \qquad \bullet(10.9a)$$

$$\frac{F_{y\,PG}}{m} = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta y} \qquad \bullet(10.9b)$$

where ΔP is the pressure change across a distance of either Δx or Δy, and ρ is the density of air.

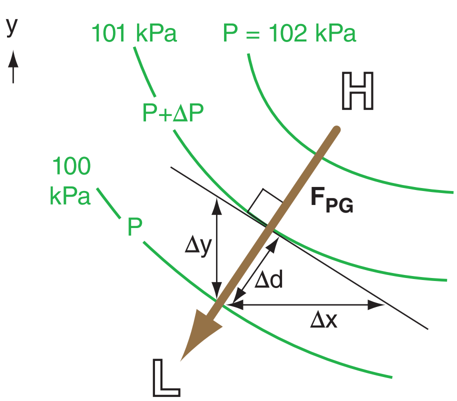


Figure 10.5
The dark arrow shows the direction of pressure-gradient force F_{PG} from high (H) to low (L) pressure. This force is perpendicular to the isobars (solid curved green lines).

Sample Application

Minneapolis (MN, USA) is about 400 km north of Des Moines (IA, USA). In Minneapolis the wind components (U, V) are (6, 4) m s^{-1}, while in Des Moines they are (2, 10) m s^{-1}. What is the value of the advective force per mass?

Find the Answer

Given: (U, V) = (6, 4) m s^{-1} in Minneapolis,
(U, V) = (2, 10) m s^{-1} in Des Moines
Δy = 400 km, Δx = is not relevant
Find: $F_{x\,AD}/m$ =? m·s^{-2}, $F_{y\,AD}/m$ =? m·s^{-2}

Use the definition of a gradient:
$\Delta U/\Delta y$ = (6 – 2 m s^{-1})/400,000 m = 1.0x10^{-5} s^{-1}
$\Delta U/\Delta x$ = not relevant, $\Delta U/\Delta z$ = not relevant,
$\Delta V/\Delta y$ = (4 – 10 m s^{-1})/400,000 m = –1.5x10^{-5} s^{-1}
$\Delta V/\Delta x$ = not relevant, $\Delta V/\Delta z$ = not relevant
Average U = (6 + 2 m s^{-1})/2 = 4 m s^{-1}
Average V = (4 + 10 m s^{-1})/2 = 7 m s^{-1}

Use eq. (10.8a):
$F_{x\,AD}/m$ = – (7m s^{-1})·(1.0x10^{-5} s^{-1})
= **–7x10**$^{-5}$ m·s^{-2}
Use eq. (10.8b):
$F_{y\,AD}/m$ = – (7m s^{-1})·(–1.5x10^{-5} s^{-1})
= **1.05x10**$^{-4}$ m·s^{-2}

Check: Physics and units are reasonable.
Exposition: The slower U winds from Des Moines are being blown by positive V winds toward Minneapolis, causing the U wind speed to decrease at Minneapolis. But the V winds are increasing there because of the faster winds in Des Moines moving northward.

Sample Application

Minneapolis (MN, USA) is about 400 km north of Des Moines (IA, USA). In (Minneapolis, Des Moines) the pressure is (101, 100) kPa. Find the pressure-gradient force per unit mass? Let ρ = 1.1 kg·m^{-3}.

Find the Answer

Given: P =101 kPa @ x = 400 km (north of Des Moines).
P =100 kPa @ x = 0 km at Des Moines. ρ = 1.1 kg·m^{-3}.
Find: $F_{y\,PG}/m$ = ? m·s^{-2}

Apply eq. (10.9b):

$$\frac{F_{y\,PG}}{m} = -\frac{1}{(1.1\text{kg}\cdot\text{m}^{-3})}\cdot\frac{(101{,}000-100{,}000)\text{Pa}}{(400{,}000-0)\text{m}}$$

= **–2.27x10**$^{-3}$ m·s^{-2}.
Hint, from Appendix A: 1 Pa = 1 kg·m^{-1}·s^{-2}.

Check: Physics and units are reasonable.
Exposition: The force is from high pressure in the north to low pressure in the south. This direction is indicated by the negative sign of the answer; namely, the force points in the negative y direction.

Sample Application

If the height of the 50 kPa pressure surface decreases by 10 m northward across a distance of 500 km, what is the pressure-gradient force?

Find the Answer

Given: $\Delta z = -10$ m, $\Delta y = 500$ km, $|g| = 9.8\ \text{m·s}^{-2}$.
Find: $F_{PG}/m = ?\ \text{m·s}^{-2}$

Use eqs. (10.11a & b):
$F_{x\,PG}/m = 0\ \text{m·s}^{-2}$, because $\Delta z/\Delta x = 0$. Thus, $F_{PG}/m = F_{y\,PG}/m$.

$$\frac{F_{y\,PG}}{m} = -|g| \cdot \frac{\Delta z}{\Delta y} = -\left(9.8\frac{\text{m}}{\text{s}^2}\right) \cdot \left(\frac{-10\text{m}}{500{,}000\text{m}}\right)$$

F_{PG}/m = $\underline{\mathbf{0.000196\ m·s^{-2}}}$

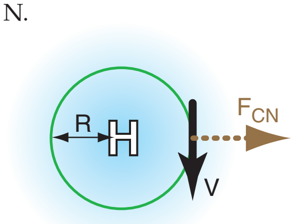
Check: Physics, units & sign are reasonable.
Exposition: For our example here, height decreases toward the north, thus a hypothetical ball would roll downhill toward the north. A northward force is in the positive *y* direction, which explains the positive sign of the answer.

Table 10-2. To apply centrifugal force to separate Cartesian coordinates, a (+/–) sign factor *s* is required.

Hemisphere	For winds encircling a Low Pressure Center	For winds encircling a High Pressure Center
Southern	–1	+1
Northern	+1	–1

Sample Application

500 km east of a high-pressure center is a north wind of 5 m s^{-1}. Assume N. Hemisphere. What is the centrifugal force?



Find the Answer

Given: $R = 5\text{x}10^5$ m,
$U = 0$, $V = -5$ m s^{-1}
Find: $F_{x\,CN}/m = ?\ \text{m·s}^{-2}$.

Apply eq. (10.13a). In Table 10-2 find $s = -1$.

$$\frac{F_{xCN}}{m} = -1 \cdot \frac{(-5\text{m/s}) \cdot (5\text{m/s})}{5 \times 10^5} \quad = \underline{\mathbf{5x10^{-5}}}\ \underline{\mathbf{m·s}}^{-2}.$$

Check: Physics and units OK. Agrees with sketch.
Exposition: To maintain a turn around the high-pressure center, other forces (the sum of which is the centripetal force) are required to pull toward the center.

If pressure increases toward one direction, then the force is in the opposite direction (from high to low *P*); hence, the negative sign in these terms.

Pressure-gradient-force magnitude is

$$\left|\frac{F_{PG}}{m}\right| = \left|\frac{1}{\rho} \cdot \frac{\Delta P}{\Delta d}\right| \tag{10.10}$$

where Δd is the distance between isobars.

Eqs. (10.9) can be rewritten using the hydrostatic eq. (1.25) to give the pressure gradient components as a function of spacing between height contours on an isobaric surface:

$$\frac{F_{x\,PG}}{m} = -|g| \cdot \frac{\Delta z}{\Delta x} \tag{10.11a}$$

$$\frac{F_{y\,PG}}{m} = -|g| \cdot \frac{\Delta z}{\Delta y} \tag{10.11b}$$

for a gravitational acceleration magnitude of $|g| = 9.8\ \text{m·s}^{-2}$. Δz is the height change in the Δx or Δy directions; hence, it is the slope of the isobaric surface. Extending this analogy of slope, if you conceptually place a ball on the isobaric surface, it will roll downhill (which is the pressure-gradient force direction). The magnitude of pressure-gradient force is

$$\left|\frac{F_{PG}}{m}\right| = \left|g \cdot \frac{\Delta z}{\Delta d}\right| \tag{10.12}$$

where Δd is distance between height contours.

The one force that makes winds blow in the horizontal is pressure-gradient force. All the other forces are a function of wind speed, hence they can only change the speed or direction of a wind that already exists. The only force that can start winds blowing from zero (calm) is pressure-gradient force.

10.3.3. Centrifugal Force

Inertia makes an air parcel try to move in a straight line. To get its path to turn requires a force in a different direction. This force, which pulls toward the inside of the turn, is called **centripetal** force. Centripetal force is the result of a net imbalance of (i.e., the nonzero vector sum of) other forces.

For mathematical convenience, we can define an apparent force, called **centrifugal** force, that is opposite to centripetal force. Namely, it points outward from the center of rotation. Centrifugal-force components are:

$$\frac{F_{x\,CN}}{m} = +s \cdot \frac{V \cdot M}{R} \qquad \bullet\text{(10.13a)}$$

$$\frac{F_{y\,CN}}{m} = -s \cdot \frac{U \cdot M}{R} \qquad \bullet\text{(10.13b)}$$

where $M = (U^2 + V^2)^{1/2}$ is wind speed (always positive), R is radius of curvature, and s is a sign factor from Table 10-2 as determined by the hemisphere (North or South) and synoptic pressure center (Low or High).

Centrifugal force magnitude is proportional to wind speed squared:

$$\left|\frac{F_{CN}}{m}\right| = \frac{M^2}{R} \tag{10.14}$$

10.3.4. Coriolis Force

An object such as an air parcel that moves relative to the Earth experiences a **compound centrifugal force** based on the combined tangential velocities of the Earth's surface and the object. When combined with the non-vertical component of gravity, the result is called Coriolis force (see the INFO box on the next page). This force points 90° to the right of the wind direction in the Northern Hemisphere (Fig. 10.6), and 90° to the left in the S. Hemisphere.

The Earth rotates one full revolution (2π radians) during a sidereal day (i.e., relative to the fixed stars, $P_{sidereal}$ is a bit less than 24 h, see Appendix B), giving an angular rotation rate of

$$\Omega = 2\cdot\pi / P_{sidereal} \qquad \bullet(10.15)$$

$$= 0.729\ 211\ 6 \times 10^{-4} \text{ radians s}^{-1}$$

The units for Ω are often abbreviated as s^{-1}. Using this rotation rate, define a **Coriolis parameter** as:

$$f_c = 2\cdot\Omega\cdot\sin(\phi) \qquad \bullet(10.16)$$

where ϕ is latitude, and $2{\cdot}\Omega = 1.458423\times10^{-4}\ s^{-1}$. Thus, the Coriolis parameter depends only on latitude. Its magnitude is roughly $1\times10^{-4}\ s^{-1}$ at mid-latitudes.

The Coriolis force in the Northern Hemisphere is:

$$\frac{F_{x\,CF}}{m} = f_c\cdot V \qquad \bullet(10.17a)$$

$$\frac{F_{y\,CF}}{m} = -f_c\cdot U \qquad \bullet(10.17b)$$

In the Southern Hemisphere the signs on the right side of eqs. (10.17) are opposite. Coriolis force is zero under calm conditions, and thus cannot create a wind. However, it can change the direction of an existing wind. Coriolis force cannot do work, because it acts perpendicular to the object's motion.

The magnitude of Coriolis force is:

$$|\ F_{CF}/m\ | \approx 2\cdot\Omega\cdot|\sin(\phi)\cdot M| \tag{10.18a}$$

or

$$|\ F_{CF}/m\ | \approx |\ f_c\cdot M\ | \tag{10.18b}$$

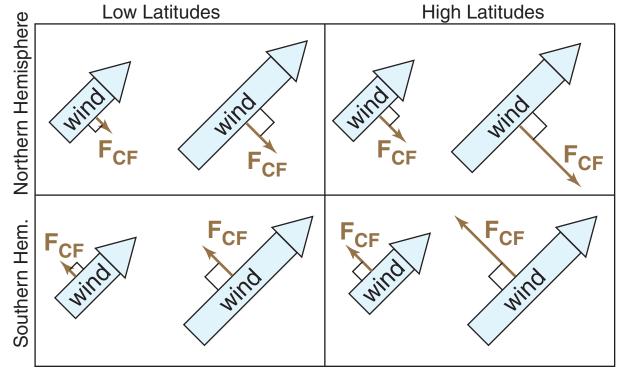
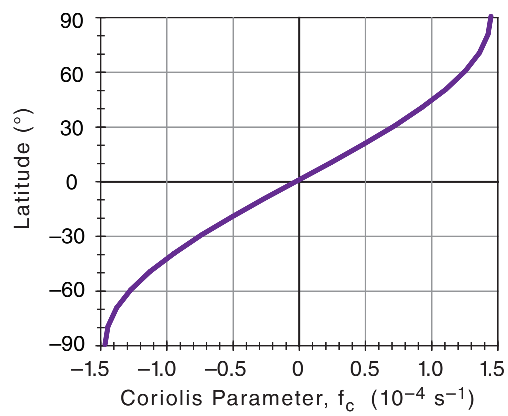


Figure 10.6
Coriolis force (F_{CF}) vs. latitude, wind-speed, and hemisphere.

Sample Application (§)
a) Plot Coriolis parameter vs. latitude.
b) Find F_{CF}/m at Paris, given a north wind of 15 m s^{-1}.

Find the Answer:
a) Given: ϕ = 48.874°N at Paris.
Find f_c (s^{-1}) vs. ϕ(°) using eq. (10.16). For example:
$f_c = (1.458\times10^{-4}\ s^{-1})\cdot\sin(48.874°) = 1.1\times10^{-4}\ s^{-1}$.



b) Given: $V = -15$ m s^{-1}. Find: F_{CF}/m= ? m s^{-2}
Assume $U = 0$ because no info, thus $F_{y\,CF}/m$= 0.
Apply eq. (10.17a):
$F_{x\,CF}/m = (1.1\times10^{-4}\ s^{-1})\cdot(-15\ m\ s^{-1})$ = **$\underline{-1.65\times10^{-3}\ m\ s^{-2}}$**
Exposition: This Coriolis force points to the west.

INFO • Coriolis Force in 3-D

Eqs. (10.17) give only the dominant components of Coriolis force. There are other smaller-magnitude Coriolis terms (labeled *small* below) that are usually neglected. The full Coriolis force in 3-dimensions is:

$$\frac{F_{x\,CF}}{m} = f_c\cdot V - 2\Omega\cdot\cos(\phi)\cdot W \tag{10.17c}$$

[*small because often W<<V*]

$$\frac{F_{y\,CF}}{m} = -f_c\cdot U \tag{10.17d}$$

$$\frac{F_{z\,CF}}{m} = 2\Omega\cdot\cos(\phi)\cdot U \tag{10.17e}$$

[*small relative to other vertical forces*]

INFO • On Coriolis Force

Gaspar Gustave Coriolis explained a compound centrifugal force on a rotating non-spherical planet such as Earth (Anders Persson: 1998, 2006, 2014).

Basics

On the rotating Earth an imbalance can occur between gravitational force and centrifugal force.

For an object of mass m moving at tangential speed M_{tan} along a curved path having radius of curvature R, **centrifugal force** was shown earlier in this chapter to be $F_{CN}/m = (M_{tan})^2/R$. In Fig 10.a the object is represented by the black dot, and the center of rotation is indicated by the X.

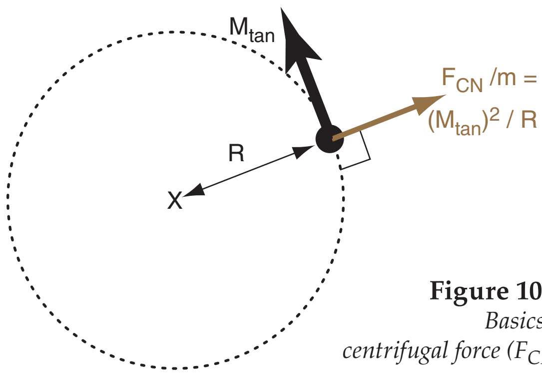


Figure 10.a. *Basics of centrifugal force (F_{CN}).*

The Earth was mostly molten early in its formation. Although gravity tends to make the Earth spherical, centrifugal force associated with Earth's rotation caused the Earth to bulge slightly at the equator. Thus, Earth's shape is an ellipsoid (Fig. 10.b).

The combination of gravity F_G and centrifugal force F_{CN} causes a net force that we feel as **effective gravity** F_{EG}. Objects fall in the direction of effective gravity, and it is how we define the local vertical (V) direction. Perpendicular to vertical is the local "horizontal" (H) direction, along the ellipsoidal surface. An object initially at rest on this surface feels no net horizontal force. [Note: Except at the poles and equator, F_G does not point exactly to Earth's center, due to gravitational pull of the equatorial bulge.]

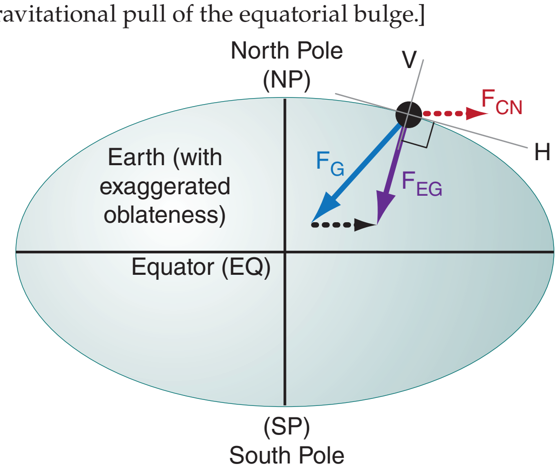


Figure 10.b. *Earth cross section (exaggerated).*

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INFO • On Coriolis Force (continuation)

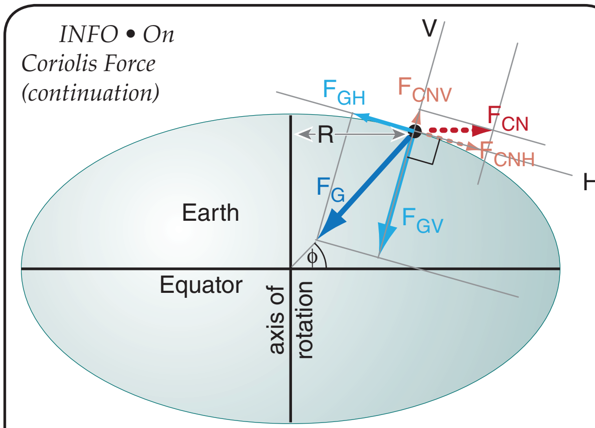

Figure 10.c. *Horizontal & vertical force components.*

Split the vectors of true gravity into local vertical F_{GV} and horizontal F_{GH} components. Do the same for the centrifugal force (F_{CNV} , F_{CNH}) of Earth's rotation (Fig. 10.c). Total centrifugal force F_{CN} is parallel to the equator (EQ). Thus, for an object at latitude ϕ , you can use trig to show $F_{CNH} \approx F_{CN} \cdot \sin(\phi)$.

Objects at Rest with respect to Earth's Surface

Looking down towards the north pole (NP), the Earth turns counterclockwise with angular velocity Ω = 360°/(sidereal day) (Fig. 10.d). Over a time interval Δt, the amount of rotation is $\Omega \cdot \Delta t$. Any object (black dot) at rest on the Earth's surface moves with the Earth at tangential speed $M_{tan} = \Omega \cdot R$ (grey arrow), where $R = R_o \cdot cos(\phi)$ is the distance from the axis of rotation. R_o = 6371 km is average Earth radius.

But because the object is at rest, its horizontal component of centrifugal force F_{CNH} associated with movement following the curved latitude (called a parallel) is the same as that for the Earth, as plotted in Fig. 10.c above. But this horizontal force is balanced by the horizontal component of gravity F_{GH}, so the object feels no net horizontal force.

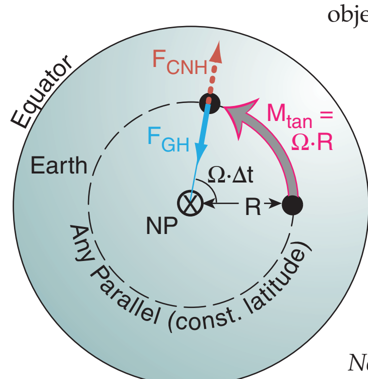

Figure 10.d. *Looking down on the North Pole (NP), for an object at rest on Earth's surface.*

Objects Moving East or West relative to Earth

Suppose an object moves with velocity M due east relative to the Earth. This velocity (thin white arrow in Fig. 10.e) is relative to Earth's velocity, giving the object a faster total velocity (grey arrow), causing greater centrifugal force and greater F_{CNH}. But F_{GH} is constant.

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INFO • On Coriolis Force (continuation)

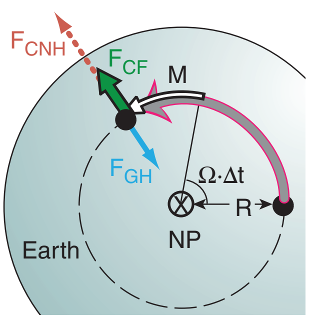


Figure 10.e.
Eastward moving object.

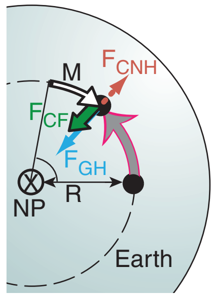


Figure 10.f.
Westward moving object.

Horizontal force F_{CNH} does NOT balance F_{GH}. The thick green arrow (Fig. 10.e) shows that the force difference F_{CF} is to the right relative to the object's motion M. F_{CF} is called **Coriolis force**.

The opposite imbalance of F_{CNH} and F_{GH} occurs for a westward-moving object (thin white arrow), because the object has slower net tangential velocity (grey arrow in Fig. 10.f). This imbalance, Coriolis force F_{CF} (green arrow), is also to the right of the relative motion M.

Northward-moving Objects

When an object moves northward at relative speed M (thin white arrow in Fig. 10.g) while the Earth is rotating, the path traveled by the object (thick grey line) has a small radius of curvature about point X that is displaced from the North Pole. The smaller radius R causes larger centrifugal force F_{CNH} pointing outward from X.

Component $F_{CNH\text{-}ns}$ of centrifugal force balances the unchanged horizontal gravitational force F_{GH}. But there remains an unbalanced east-west component of centrifugal force $F_{CNH\text{-}ew}$ which is defined as Coriolis force F_{CF} (green arrow). Again, it is to the right of the relative motion vector M of the object.

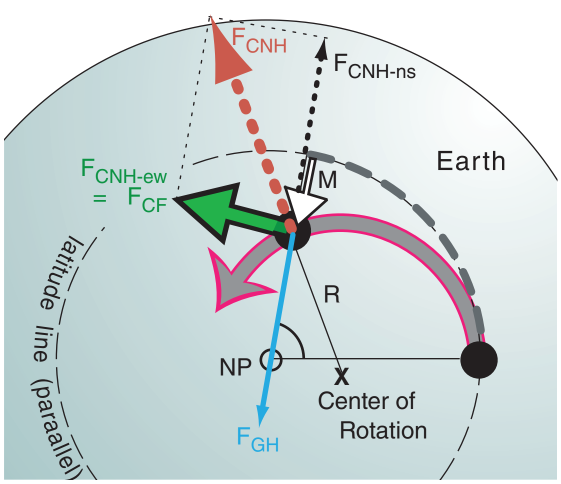


Figure 10.g. *Northward moving object.*

(continues in next column)

INFO • Coriolis Force (continuation)

Objects moving south have a Coriolis force to the right due to the larger radius of curvature. Regardless of the direction of motion in the Northern Hemisphere, Coriolis force acts 90° to the right of the object's motion relative to the Earth. When viewing the Southern Hemisphere from below the south pole, the Earth rotates clockwise, causing a Coriolis force that is 90° to the left of the relative motion vector.

Coriolis-force Magnitude Derivation

From Figs. 10.c & d, see that an object at rest (subscript R) has

$$F_{GH} = F_{CNH} \equiv F_{CNHR} \qquad \text{(C1)}$$

and

$$M_{tan\ rest} = \Omega \cdot R \qquad \text{(C2)}$$

From Fig. 10.e, Coriolis force for an eastward-moving object is defined as

$$F_{CF} \equiv F_{CNH} - F_{GH}$$

Apply eq. (C1) to get

$$F_{CF} = F_{CNH} - F_{CNHR}$$

or

$$F_{CF} = \sin(\phi) \cdot [F_{CN} - F_{CNR}] \qquad \text{(from Fig. 10.c)}$$

Divide by mass m, and plug in the definition for centrifugal force as velocity squared divided by radius:

$$F_{CF} / m = \sin(\phi) \cdot [(M_{tan})^2/R - (M_{tan\ rest})^2/R]$$

Use $M_{tan} = M_{tan\ rest} + M$, along with eq. (C2):

$$F_{CF} / m = \sin(\phi) \cdot [(\Omega{\cdot}R+M)^2/R - (\Omega{\cdot}R)^2/R]$$

$$F_{CF} / m = \sin(\phi) \cdot [(2{\cdot}\Omega{\cdot}M) + (M^2/R)]$$

The first term is usually much larger than the last, allowing the following approximation for Coriolis force per mass:

$$F_{CF} / m \approx 2{\cdot}\Omega{\cdot}\sin(\phi) \cdot M \qquad (10.18)$$

Define a Coriolis parameter as $f_c \equiv 2{\cdot}\Omega{\cdot}\sin(\phi)$. Thus,

$$F_{CF} / m \approx f_c \cdot M$$

HIGHER MATH • Apparent Forces

In vector form, centrifugal force/mass for an object at rest on Earth is $-\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$, and Coriolis force/mass is $-2\mathbf{\Omega} \times \mathbf{V}$, where vector $\mathbf{\Omega}$ points along the Earth's axis toward the north pole, $\mathbf{r}$ points from the Earth's center to the object, $\mathbf{V}$ is the object's velocity relative to Earth, and $\times$ is the vector cross product.

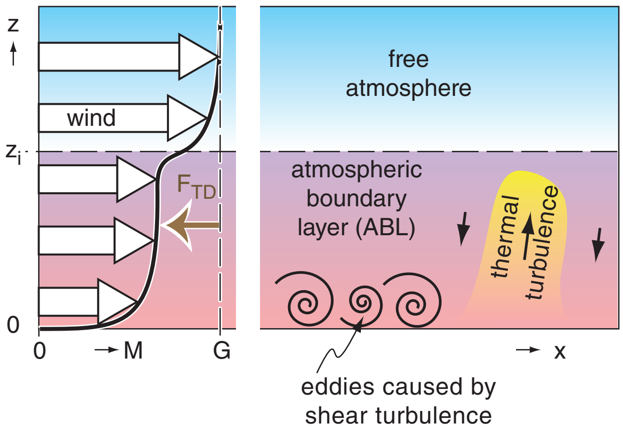


Figure 10.7
Wind speed M (curved black line with white highlights) is slower than geostrophic G (vertical dashed line) because of turbulent drag force F_{TD} in the atmospheric boundary layer.

Sample Application
What is the drag force per unit mass opposing a U = 15 m s^{-1} wind (with V = 0) for a: (a) statically neutral ABL over a rough forest; & (b) statically unstable ABL having convection with w_B = 50 m s^{-1}, given z_i = 1.5 km.

Find the Answer
Given: $U = M = 15$ m s^{-1}, $z_i = 1500$ m,
$C_D = 2\text{x}10^{-2}$, $w_B = 50$ m s^{-1}.
Find: $F_{x\,TD}/m$ = ? m·s^{-2}.

(a) Plugging eq. (10.21) into eq. (10.19a) gives:

$$\frac{F_{xTD}}{m} = -C_D \cdot M \cdot \frac{U}{z_i} = -(0.02)\cdot\frac{(15\text{m/s})^2}{1500\text{m}}$$

$$= \underline{-3\text{x}10^{-3}} \quad \text{m·s}^{-2}.$$

(b) Plugging eq. (10.22) into eq. (10.19a) gives:

$$\frac{F_{xTD}}{m} = -b_D \cdot w_B \cdot \frac{U}{z_i}$$

$$= -(0.00183)\cdot(50\text{m/s})\cdot\frac{(15\text{m/s})}{1500\text{m}}$$

$$= \underline{-9.15\text{x}10^{-4}} \quad \text{m·s}^{-2}.$$

Check: Physics and units are reasonable.
Exposition: Because the wind is positive (blowing toward the east) it requires that the drag be negative (pushing toward the west). Shear (mechanical) turbulence and convective (thermal/buoyant) turbulence can both cause drag by diluting the faster winds higher in the ABL with slower near-surface winds.

10.3.5. Turbulent-Drag Force

Surface elements such as pebbles, blades of grass, crops, trees, and buildings partially block the wind, and disturb the air that flows around them. The combined effect of these elements over an area of ground is to cause resistance to air flow, thereby slowing the wind. This resistance is called **drag**.

At the bottom of the troposphere is a layer of air roughly 0.3 to 3 km thick called the **atmospheric boundary layer** (**ABL**). The ABL is named because it is at the bottom boundary of the atmosphere. Turbulence in the ABL mixes the very-slow near-surface air with the faster air in the ABL, reducing the wind speed M throughout the entire ABL (Fig. 10.7).

The net result is a drag force that is normally only felt by air in the ABL. For ABL depth z_i the drag is:

$$\frac{F_{x\,TD}}{m} = -w_T \cdot \frac{U}{z_i} \qquad \bullet\text{(10.19a)}$$

$$\frac{F_{y\,TD}}{m} = -w_T \cdot \frac{V}{z_i} \qquad \bullet\text{(10.19b)}$$

where w_T is called a turbulent **transport velocity**.

The total magnitude of turbulent drag force is

$$\left|\frac{F_{TD}}{m}\right| = w_T \cdot \frac{M}{z_i} \qquad \text{(10.20)}$$

and is always opposite to the wind direction.

For statically **unstable** ABLs with light winds, where a warm underlying surface causes thermals of warm buoyant air to rise (Fig. 10.7), this convective turbulence transports drag information upward at rate:

$$w_T = b_D \cdot w_B \qquad \text{(10.22)}$$

where dimensionless factor b_D = 1.83x10^{-3}. The **buoyancy velocity scale**, w_B, is of order 10 to 50 m s^{-1}, as is explained in the Heat Budget chapter.

For statically **neutral** conditions where strong winds M and **wind shears** (changes of wind direction and/or speed with height) create eddies and mechanical turbulence near the ground (Fig. 10.7), the transport velocity is

$$w_T = C_D \cdot M \qquad \text{(10.21)}$$

where the **drag coefficient** C_D is small (2x10^{-3} dimensionless) over smooth surfaces and is larger (2x10^{-2}) over rougher surfaces such as forests.

In fair weather, turbulent-drag force is felt only in the ABL. However, thunderstorm turbulence can mix slow near-surface air throughout the troposphere. Fast winds over mountains can create mountain-wave drag felt in the whole atmosphere (see the Regional Winds chapter).

10.3.6. Summary of Forces

Table 10-3. Summary of forces.

Item	Name of Force	Direction	Magnitude (N kg^{-1})	Horiz. (H) or Vert. (V)	Remarks ("item" is in column 1; H & V in col. 5)
1	**gravity**	down	$\left\|\frac{F_G}{m}\right\| = \|g\| = 9.8\ \text{m·s}^{-2}$	V	**hydrostatic equilibrium** when items 1 & 2V balance
2	**pressure gradient**	from high to low pressure	$\left\|\frac{F_{PG}}{m}\right\| = \left\|g \cdot \frac{\Delta z}{\Delta d}\right\|$	V & H	the only force that can drive horizontal winds
3	**Coriolis** (compound)	90° to right (left) of wind in Northern (Southern) Hemisphere	$\left\|\frac{F_{CF}}{m}\right\| = 2 \cdot \Omega \cdot \|\sin(\phi) \cdot M\|$	H*	**geostrophic wind** when 2H and 3 balance (explained later in horiz. wind section)
4	**turbulent drag**	opposite to wind	$\left\|\frac{F_{TD}}{m}\right\| = w_T \cdot \frac{M}{z_i}$	H*	**atm. boundary-layer wind** when 2H, 3 and 4 balance (explained in horiz. wind section)
5	**centrifugal** (apparent)	away from center of curvature	$\left\|\frac{F_{CN}}{m}\right\| = \frac{M^2}{R}$	H*	centripetal = opposite of centrifugal. **Gradient wind** when 2H, 3 and 5 balance
6	**advection** (apparent)	(any)	$\left\|\frac{F_{AD}}{m}\right\| = \left\|-M \cdot \frac{\Delta U}{\Delta d} - \dots\right\|$	V & H	neither creates nor destroys momentum; just moves it

Horizontal is the direction we will focus on. However, Coriolis force has a small vertical component for zonal winds. Turbulent drag can exist in the vertical for rising or sinking air, but has completely different form than the boundary-layer drag given above. Centrifugal force can exist in the vertical for vortices with horizontal axes. Note: units N kg^{-1} = m·s^{-2}.

10.4. EQUATIONS OF HORIZONTAL MOTION

Combining the forces from eqs. (10.7, 10.8, 10.9, 10.17, and 10.19) into Newton's Second Law of Motion (eq. 10.5) gives simplified equations of horizontal motion:

•(10.23a)

$$\frac{\Delta U}{\Delta t} = -U\frac{\Delta U}{\Delta x} - V\frac{\Delta U}{\Delta y} - W\frac{\Delta U}{\Delta z} - \frac{1}{\rho}\cdot\frac{\Delta P}{\Delta x} + f_c \cdot V - w_T \cdot \frac{U}{z_i}$$

•(10.23b)

$$\underbrace{\frac{\Delta V}{\Delta t}}_{\text{tendency}} = \underbrace{-U\frac{\Delta V}{\Delta x} - V\frac{\Delta V}{\Delta y} - W\frac{\Delta V}{\Delta z}}_{\text{advection}} \underbrace{- \frac{1}{\rho}\cdot\frac{\Delta P}{\Delta y}}_{\text{pressure gradient}} \underbrace{- f_c \cdot U}_{\text{Coriolis}} \underbrace{- w_T \cdot \frac{V}{z_i}}_{\text{turbulent drag}}$$

These are the forecast equations for wind.

For special conditions where steady winds around a circle are anticipated, centrifugal force can be included.

The terms on the right side of eqs. (10.23) can all be of order $1\text{x}10^{-4}$ to $10\text{x}10^{-4}$ m·s^{-2} (which is equivalent to units of N kg^{-1}, see Appendix A for review). However, some of the terms can be neglected under special conditions where the flow is less complicated. For example, near-zero Coriolis force occurs near the equator. Near-zero turbulent drag exists above the ABL. Near-zero pressure gradient is at low- and high-pressure centers.

Other situations are more complicated, for which additional terms should be added to the equations of horizontal motion. Within a few mm of the ground, **molecular friction** is large. Above mountains during windy conditions, **mountain-wave drag** is large. Above the ABL, cumulus clouds and thunderstorms can create strong **convective mixing**.

For a few idealized situations where many terms in the equations of motion are small, it is possible to solve those equations for the horizontal wind speeds. These theoretical winds are presented in the next section. Later in this chapter, equations to forecast vertical motion (W) will be presented.

Table 10-4. Names of idealized steady-state horizontal winds, and the forces that govern them.

$$0 = -U\frac{\Delta U}{\Delta x} \quad -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta x} \quad +f_c\cdot V \quad -w_T\cdot\frac{U}{z_i} \quad +s\frac{V\cdot M}{R}$$

Forces:	*pressure gradient*	*Coriolis*	*turbulent drag*	*centri-fugal*
Wind Name				
Geostrophic	•	•		
Gradient	•	•		•
Atm.Bound. Layer	•	•	•	
ABL Gradient	•	•	•	•
Cyclostrophic	•			•
Inertial		•		•
Antitriptic	•		•	

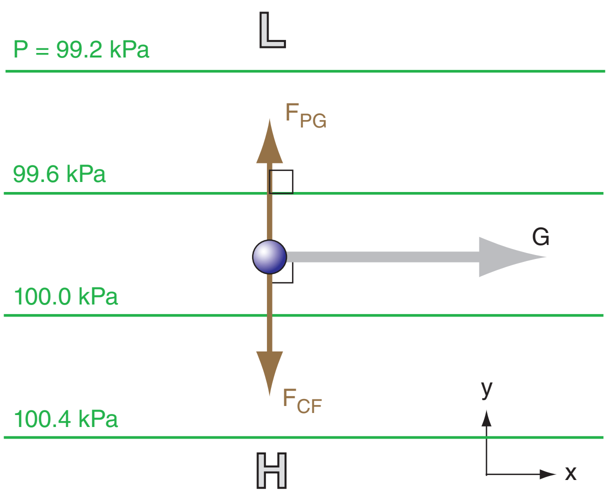


Figure 10.8
Idealized weather map for the Northern Hemisphere, showing geostrophic wind (G, grey arrow) caused by a balance between two forces (brown arrows): pressure-gradient force (F_{PG}) and Coriolis force (F_{CF}). P is pressure, with isobars plotted as thin green lines. L and H are low and high-pressure regions. The small sphere represents an air parcel.

Sample Application

Find geostrophic wind components at a location where ρ = 1.2 kg m^{-3} and f_c = 1.1x10^{-4} s^{-1}. Pressure decreases by 2 kPa for each 800 km of distance north.

Find the Answer

Given: f_c =1.1x10^{-4} s^{-1}, ΔP= –2 kPa, ρ=1.2 kg m^{-3}, Δy=800 km.
Find: (U_g, V_g) = ? m s^{-1}

But $\Delta P/\Delta x = 0$ implies $V_g = 0$. For U_g, use eq. (10.26a):

$$U_g = \frac{-1}{(1.2\text{kg/m}^3)\cdot(1.1\times10^{-4}\text{s}^{-1})}\cdot\frac{(-2\text{kPa})}{(800\text{km})} = \underline{\mathbf{18.9\ m\ s^{-1}}}$$

Check: Physics & units OK. Agrees with Fig. 10.10.
Exposition: As the pressure gradient accelerates air northward, Coriolis force turns it toward the east.

10.5. HORIZONTAL WINDS

When air accelerates to create wind, forces that are a function of wind speed also change. As the winds continue to accelerate under the combined action of all the changing forces, **feedbacks** often occur to eventually reach a final wind where the forces balance. With a zero net force, there is zero acceleration.

Such a final, equilibrium, state is called **steady state**:

$$\frac{\Delta U}{\Delta t} = 0, \quad \frac{\Delta V}{\Delta t} = 0 \qquad \bullet(10.24)$$

Caution: Steady state means no further change to the <u>non-zero winds</u>. Do not assume the winds are zero.

Under certain idealized conditions, some of the forces in the equations of motion are small enough to be neglected. For these situations, theoretical steady-state winds can be found based on only the remaining larger-magnitude forces. These theoretical winds are given special names, as listed in Table 10-4. These winds are examined next in more detail. As we discuss each theoretical wind, we will learn where we can expect these in the real atmosphere.

10.5.1. Geostrophic Wind

For special conditions where the only forces are Coriolis and pressure-gradient (Fig. 10.8), the resulting steady-state wind is called the geostrophic wind, with components (U_g, V_g). For this special case, the only terms remaining in, eqs. (10.23) are:

$$0 = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta x} + f_c\cdot V \qquad (10.25a)$$

$$0 = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta y} - f_c\cdot U \qquad (10.25b)$$

Define $U \equiv U_g$ and $V \equiv V_g$ in the equations above, and then solve for these wind components:

$$U_g = -\frac{1}{\rho\cdot f_c}\cdot\frac{\Delta P}{\Delta y} \qquad \bullet(10.26a)$$

$$V_g = +\frac{1}{\rho\cdot f_c}\cdot\frac{\Delta P}{\Delta x} \qquad \bullet(10.26b)$$

where f_c = (1.4584x10^{-4} s^{-1})·sin(*latitude*) is the Coriolis parameter, ρ is air density, and $\Delta P/\Delta x$ and $\Delta P/\Delta y$ are the horizontal pressure gradients.

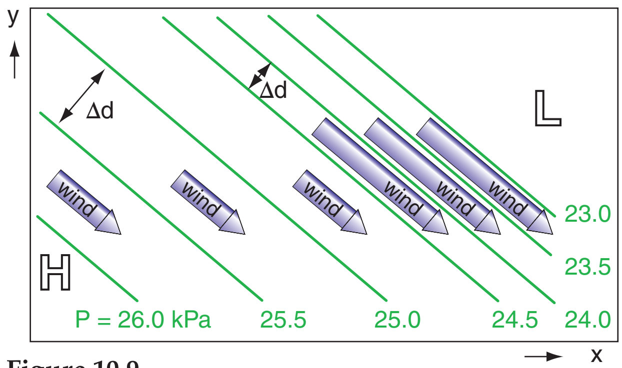


Figure 10.9
Isobars (green lines) that are more closely spaced (i.e., tightly packed) cause stronger geostrophic winds (arrows), for N. Hemisphere.

Real winds are nearly geostrophic at locations where isobars or height contours are relatively straight, for altitudes above the atmospheric boundary layer. Geostrophic winds are fast where isobars are packed closer together. The geostrophic wind direction is parallel to the height contours or isobars. In the N. (S.) hemisphere the wind direction is such that low pressure is to the wind's left (right), see Fig. 10.9.

The magnitude G of the geostrophic wind is:

$$G = \sqrt{U_g^{\,2} + V_g^{\,2}} \qquad (10.27)$$

If Δd is the distance between two isobars (in the direction of greatest pressure change; namely, perpendicular to the isobars), then the magnitude (Fig. 10.10) of the geostrophic wind is:

$$G = \left| \frac{1}{\rho \cdot f_c} \cdot \frac{\Delta P}{\Delta d} \right| \qquad \bullet(10.28)$$

Above sea level, weather maps are often on isobaric surfaces (constant pressure charts), from which the geostrophic wind (Fig. 10.10) can be found from the height gradient (change of height of the isobaric surface with horizontal distance):

$$U_g = -\frac{|g|}{f_c} \cdot \frac{\Delta z}{\Delta y} \qquad \bullet(10.29a)$$

$$V_g = +\frac{|g|}{f_c} \cdot \frac{\Delta z}{\Delta x} \qquad \bullet(10.29b)$$

where the Coriolis parameter is f_c, and gravitational acceleration is $|g|$ = 9.8 m·s^{-2}. The corresponding magnitude of geostrophic wind on an isobaric chart is:

$$G = \left| \frac{g}{f_c} \cdot \frac{\Delta z}{\Delta d} \right| \qquad \bullet(10.29c)$$

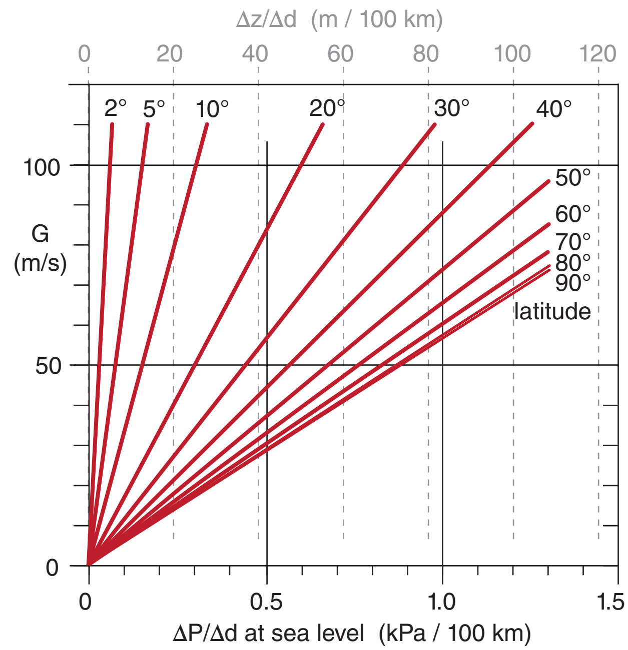


Figure 10.10
Variation of geostrophic wind speed (G) with horizontal pressure gradient (ΔP/Δd) at sea level. Top scale is height gradient of any isobaric surface.

Sample Application
Find the geostrophic wind for a height increase of 50 m per 200 km of distance toward the east. Assume, $f_c = 0.9\text{x}10^{-4}\ \text{s}^{-1}$.

Find the Answer
Given: Δx = 200 km, Δz = 50 m, $f_c = 0.9\text{x}10^{-4}\ \text{s}^{-1}$.
Find: G = ? m s^{-1}

No north-south height gradient, thus $U_g = 0$.
Apply eq. (10.29b) and set $G = V_g$:

$$V_g = +\frac{|g|}{f_c} \cdot \frac{\Delta z}{\Delta x} = \left(\frac{9.8\text{m s}^{-2}}{0.00009\text{s}^{-1}} \right) \cdot \left(\frac{50\text{m}}{200{,}000\text{m}} \right) = \underline{\mathbf{27.2\ m\ s^{-1}}}$$

Check: Physics & units OK. Agrees with Fig. 10.10.
Exposition: If height increases towards the east, then you can imagine that a ball placed on such a surface would roll downhill toward the west, but would turn to its right (toward the north) due to Coriolis force.

INFO • Approach to Geostrophy

How does an air parcel, starting from rest, approach the final steady-state geostrophic wind speed G sketched in Fig. 10.8?

Start with the equations of horizontal motion (10.23), and ignore all terms except the tendency, pressure-gradient force, and Coriolis force. Use the definition of

continues on next page

INFO • Appr. to Geostrophy *(continuation)*

geostrophic wind (eqs. 10.26) to write the resulting simplified equations as:

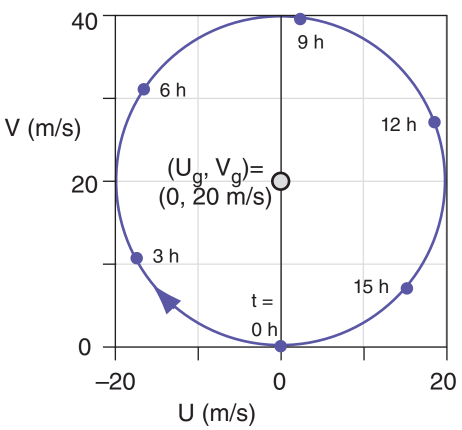
$$\Delta U / \Delta t = -f_c \cdot (V_g - V)$$
$$\Delta V / \Delta t = f_c \cdot (U_g - U)$$

Next, rewrite these as forecast equations:

$$U_{new} = U_{old} - \Delta t \cdot f_c \cdot (V_g - V_{old})$$
$$V_{new} = V_{old} + \Delta t \cdot f_c \cdot (U_g - U_{new})$$

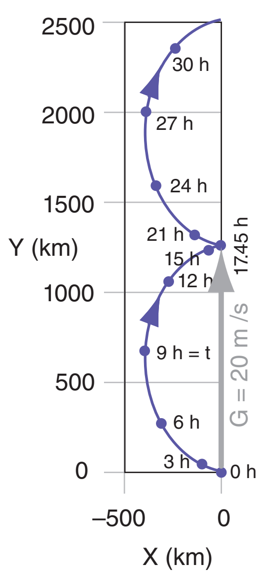
Start with initial conditions (U_{old}, V_{old}) = (0, 0), and then iteratively solve the equations on a spreadsheet to forecast the wind.

For example, suppose ΔP = 1 kPa, $f_c = 10^{-4}$ s^{-1}, Δx = 500 km, ρ = 1 kg m^{-3}, where we would anticipate the wind should approach (U_g, V_g) = (0, 20) m s^{-1}. The actual evolution of winds (U, V) and air parcel position (X, Y) are shown in Figs. below.



Surprisingly, the winds never reach geostrophic equilibrium, but instead rotate around the geostrophic wind. This is called an **inertial oscillation**, with period of $2 \cdot \pi / f_c$. For our case, the period is 17.45 h. Twice this period is called a **pendulum day**.

The net result in the figure below is that the wind indeed moves at the geostrophic speed of 20 m s^{-1} to the north (≈ 1250 km in 17.45 h), but along the way it staggers west and east with an additional **ageostrophic** (non-geostrophic) part.



Inertial oscillations are sometimes observed at night in the atmospheric boundary layer, but rarely higher in the atmosphere. Why not? (1) The ageostrophic component of wind (wind from the East in this example) moves air mass, and changes the pressure gradient. (2) Friction damps the oscillation toward a steady wind.

If the **geopotential** $\Phi = |g| \cdot z$ is substituted in eqs. (10.29), the resulting geostrophic winds are:

$$U_g = -\frac{1}{f_c} \cdot \frac{\Delta \Phi}{\Delta y} \tag{10.30a}$$

$$V_g = \frac{1}{f_c} \cdot \frac{\Delta \Phi}{\Delta x} \tag{10.30b}$$

10.5.2. Gradient Wind

If there is no turbulent drag, then winds tend to blow parallel to isobar lines or height-contour lines even if those lines are curved. However, if the lines curve around a low-pressure center (in either hemisphere), then the wind speeds are **subgeostrophic** (i.e., slower than the theoretical geostrophic wind speed). For lines curving around high-pressure centers, wind speeds are **supergeostrophic** (faster than theoretical geostrophic winds). These theoretical winds following curved isobars or height contours are known as **gradient winds**.

Gradient winds differ from geostrophic winds because Coriolis force F_{CF} and pressure-gradient force F_{PG} do not balance, resulting in a non-zero net force F_{net}. This net force is called centripetal force, and is what causes the wind to continually change direction as it goes around a circle (Figs. 10.11 & 10.12). By describing this change in direction as causing an apparent force (centrifugal), we can find the equations that define a steady-state gradient wind:

$$0 = \underbrace{-\frac{1}{\rho} \cdot \frac{\Delta P}{\Delta x}}_{\text{pressure gradient}} \underbrace{+ f_c \cdot V}_{\text{Coriolis}} \underbrace{+ s \cdot \frac{V \cdot M}{R}}_{\text{centrifugal}} \tag{10.31a}$$

$$0 = \underbrace{-\frac{1}{\rho} \cdot \frac{\Delta P}{\Delta y}}_{\text{pressure gradient}} \underbrace{- f_c \cdot U}_{\text{Coriolis}} \underbrace{- s \cdot \frac{U \cdot M}{R}}_{\text{centrifugal}} \tag{10.31b}$$

Because the gradient wind is for flow around a circle, we can frame the governing equations in radial coordinates, such as for flow around a low:

$$\frac{1}{\rho} \cdot \frac{\Delta P}{\Delta R} = f_c \cdot M_{tan} + \frac{M_{tan}^2}{R} \tag{10.32}$$

where R is radial distance from the center of the circle, f_c is the Coriolis parameter, ρ is air density, $\Delta P / \Delta R$ is the radial pressure gradient, and M_{tan} is the magnitude of the tangential velocity; namely, the gradient wind.

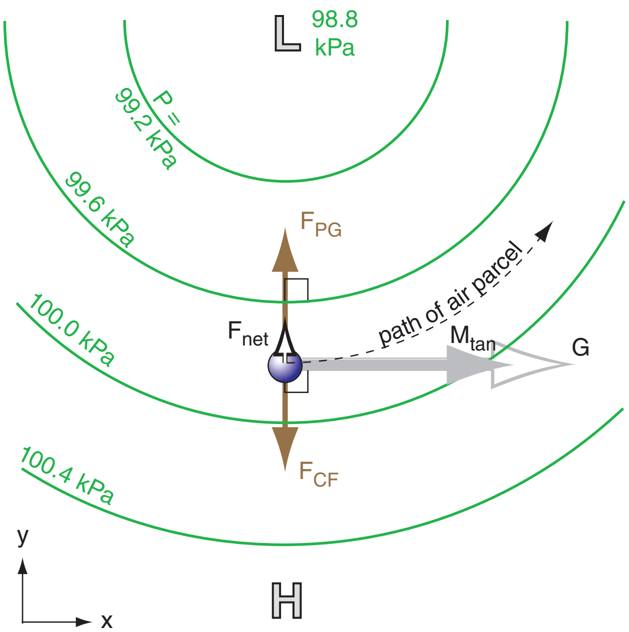


Figure 10.11
Forces (brown arrows) that cause the gradient wind (solid grey arrow, M_{tan}) to be slower than geostrophic (hollow grey arrow) when circling around a low-pressure center (called a cyclone in the N. Hem.). The short white arrow with black outline shows centripetal force (the imbalance between the other two forces). Centripetal force pulls the air parcel (blue sphere) inward to force the wind direction to change as needed for the wind to turn along a circular path.

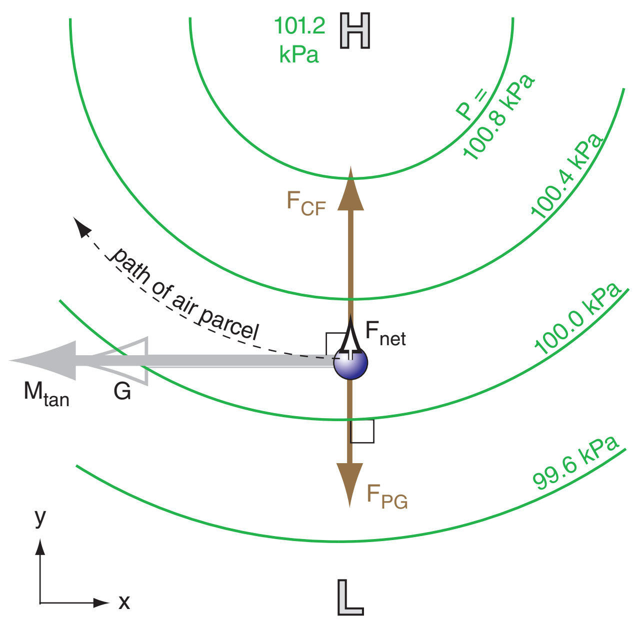


Figure 10.12
Forces (brown arrows) that cause the gradient wind (solid grey arrow, M_{tan}) to be faster than geostrophic (hollow grey arrow) for an air parcel (blue sphere) circling around a high-pressure center (called an anticyclone in the N. Hemisphere).

By re-arranging eq. (10.32) and plugging in the definition for geostrophic wind speed G, you can get an implicit solution for the gradient wind M_{tan}:

$$M_{tan} = G \pm \frac{M_{tan}{}^2}{f_c \cdot R} \tag{10.33}$$

In this equation, use the + sign for flow around high-pressure centers, and the – sign for flow around lows (Fig. 10.13).

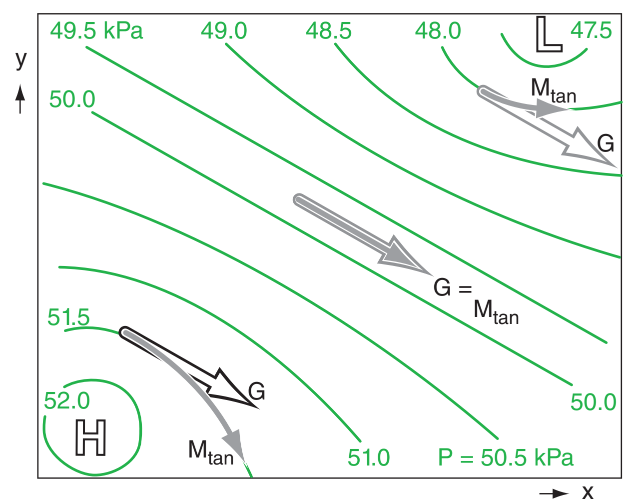


Figure 10.13
Comparison of gradient winds M_{tan} vs. geostrophic wind G for flows around low (L) and high (H) pressures. N. Hemisphere.

Sample Application
What radius of curvature causes the gradient wind to equal the geostrophic wind?

Find the Answer
Given: $M_{tan} = G$ Find: R = ? km

Use eq. (10.33), with $M_{tan} = G$: $G = G \pm G^2/(f_c{\cdot}R)$
This is a valid equality $G = G$ only when the last term in eq. (10.33) approaches zero; i.e., in the limit of $\underline{\mathbf{R = \infty}}$.

Check: Eq. (10.33) still balances in this limit. **Exposition**: Infinite radius of curvature is a straight line, which (in the absence of any other forces such as turbulent drag) is the condition for geostrophic wind.

INFO • The Rossby Number

The **Rossby number** (Ro) is a dimensionless ratio defined by

$$Ro = \frac{M}{f_c \cdot L} \qquad \text{or} \qquad Ro = \frac{M}{f_c \cdot R}$$

where M is wind speed, f_c is the Coriolis parameter, L is a characteristic length scale, and R is radius of curvature.

In the equations of motion, suppose that advection terms such as $U \cdot \Delta U/\Delta x$ are order of magnitude M^2/L, and Coriolis terms are of order $f_c \cdot M$. Then the Rossby number is like the ratio of advection to Coriolis terms: $(M^2/L) \,/\, (f_c \cdot M) = M/(f_c \cdot L) = Ro$. Or, we could consider the Rossby number as the ratio of centrifugal (order of M^2/R) to Coriolis terms, yielding $M/(f_c \cdot R) = Ro$.

Use the Rossby number as follows. If $Ro < 1$, then Coriolis force is a dominant force, and the flow tends to become geostrophic (or gradient, for curved flow). If $Ro > 1$, then the flow tends not to be geostrophic.

For example, a midlatitude cyclone (low-pressure system) has approximately $M = 10$ m s^{-1}, $f_c = 10^{-4}$ s^{-1}, and $R = 1000$ km, which gives $Ro = 0.1$. Hence, midlatitude cyclones tend to adjust toward geostrophic balance, because $Ro < 1$. In contrast, a tornado has roughly $M = 50$ m s^{-1}, $f_c = 10^{-4}$ s^{-1}, and $R = 50$ m, which gives $Ro = 10{,}000$, which is so much greater than one that geostrophic balance is not relevant.

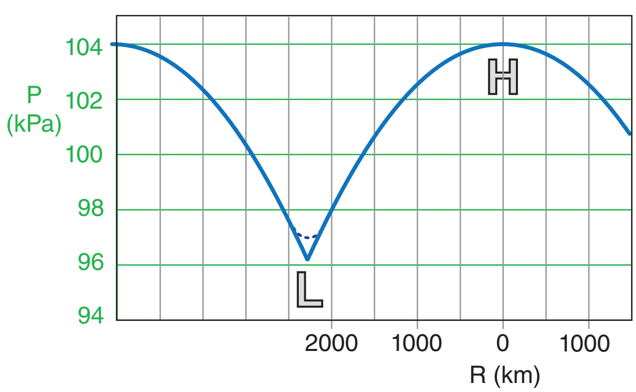


Figure 10.14
Illustration of how mean sea-level pressure P can vary with distance R from a high-pressure (ℍ) center. The anticyclone (i.e., the high) has zero horizontal pressure gradient and calm winds in its center, with weak pressure gradient (ΔP/ΔR) and gentle winds in a broad region around it. The cyclone (i.e., the low) can have steep pressure gradients and associated strong winds close to the low center (𝕃), with a pressure cusp right at the low center. In reality (dotted line), turbulent mixing near the low center smooths the cusp, allowing a small region of light winds at the low center surrounded by stronger winds. Although this graph was constructed using eq. (10.37b), it approximates the pressure variation along the cross section shown in the next figure.

Eq. (10.33) is a quadratic equation that has two solutions. One solution is for the gradient wind M_{tan} around a **cyclone** (i.e., a low):

$$M_{tan} = 0.5 \cdot f_c \cdot R \cdot \left[-1 + \sqrt{1 + \frac{4 \cdot G}{f_c \cdot R}} \right] \qquad \bullet(10.34a)$$

The other solution is for flow around an **anticyclone** (i.e., a high):

$$M_{tan} = 0.5 \cdot f_c \cdot R \cdot \left[1 - \sqrt{1 - \frac{4 \cdot G}{f_c \cdot R}} \right] \qquad \bullet(10.34b)$$

To simplify the notation in the equations above, let

$$Ro_c = \frac{G}{f_c \cdot R} \qquad (10.35)$$

where we can identify (Ro_c) as a "curvature" **Rossby number** because its length scale is the radius of curvature (R). When Ro_c is small, the winds are roughly geostrophic; namely, pressure gradient force nearly balances Coriolis force. [CAUTION: In later chapters you will learn about a Rossby radius of deformation, which is distinct from both Ro_c and R.]

For winds blowing around a <u>low</u>, the gradient wind is:

$$M_{tan} = \frac{G}{2 \cdot Ro_c} \cdot \left[-1 + \left(1 + 4 \cdot Ro_c\right)^{1/2} \right] \qquad (10.36a)$$

and for winds around a <u>high</u>) the gradient wind is:

$$M_{tan} = \frac{G}{2 \cdot Ro_c} \cdot \left[1 - \left(1 - 4 \cdot Ro_c\right)^{1/2} \right] \qquad (10.36b)$$

where G is the geostrophic wind.

While the differences between solutions (10.36a & b) appear subtle at first glance, these differences have a significant impact on the range of winds that are physically possible. Any value of Ro_c can yield physically reasonable winds around a <u>low</u>-pressure center (eq. 10.36a). But to maintain a positive argument inside the square root of eq. (10.36b), only values of $Ro_c \le 1/4$ are allowed for a <u>high</u>.

Thus, strong radial pressure gradients with small radii of curvature, and strong tangential winds can exist near low center. But only weak pressure gradients with large radii of curvature and light winds are possible near high-pressure centers (Figs. 10.14 and 10.15). To find the maximum allowable horizontal variations of height z or pressure P near anticyclones, use $Ro_c = 1/4$ in eq. (10.35) with G from (10.29c) or (10.28):

$$z = z_c - \left(f_c^{\,2} \cdot R^2\right) / \left(8 \cdot |g|\right) \qquad \bullet(10.37a)$$

or

$$P = P_c - \left(\rho \cdot f_c^{\,2} \cdot R^2\right) / 8 \qquad \bullet(10.37b)$$

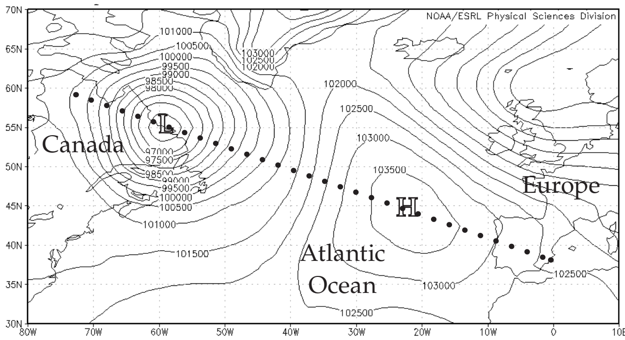


Figure 10.15
Illustration of strong pressure gradients (closely-spaced isobars) around the low-pressure center (L) over eastern Canada, and weak pressure gradients (isobars spaced further apart) around the high (H) over the NE Atlantic Ocean. NCEP reanalysis of daily-average mean sea-level pressure (Pa) for 5 Feb 2013. Pressures in the low & high centers were 96.11 & 104.05 kPa. Pressure variation along the dotted line is similar to that plotted in the previous figure. [Courtesy of the NOAA/NCEP Earth Systems Research Laboratory. http://www.esrl.noaa.gov/psd/data/gridded/data.ncep.reanalysis.html]

where the center pressure in the high (anticyclone) is P_c , or for an isobaric surface the center height is z_c, the Coriolis parameter is f_c , $|g|$ is gravitational acceleration magnitude, ρ is air density, and the radius from the center of the high is R (see Fig. 10.14).

Figs. 10.14 and 10.15 show that pressure gradients, and thus the geostrophic wind, can be large near low centers. However, pressure gradients, and thus the geostrophic wind, must be small near high centers. This difference in geostrophic wind speed G between lows and highs is sketched in Fig. 10.16. The slowdown of gradient wind M_{tan} (relative to geostrophic) around lows, and the speedup of gradient wind (relative to geostrophic) around highs is also plotted in Fig. 10.16. The net result is that gradient winds, and even atmospheric boundary-layer gradient winds M_{ABLG} (described later in this chapter), are usually stronger (in an absolute sense) around lows than highs. For this reason, low-pressure centers are often windy.

10.5.3. Atmospheric-Boundary-Layer Wind

If you add turbulent drag to winds that would have been geostrophic, the result is a **subgeostrophic** (slower-than-geostrophic) wind that crosses the isobars at angle (α) (Fig. 10.17). This condition is found in the atmospheric boundary layer (ABL) where the isobars are straight. The force balance at steady state is:

$$0 = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta x} + f_c \cdot V - w_T \cdot \frac{U}{z_i} \qquad (10.38a)$$

$$0 = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta y} - f_c \cdot U - w_T \cdot \frac{V}{z_i} \qquad (10.38b)$$

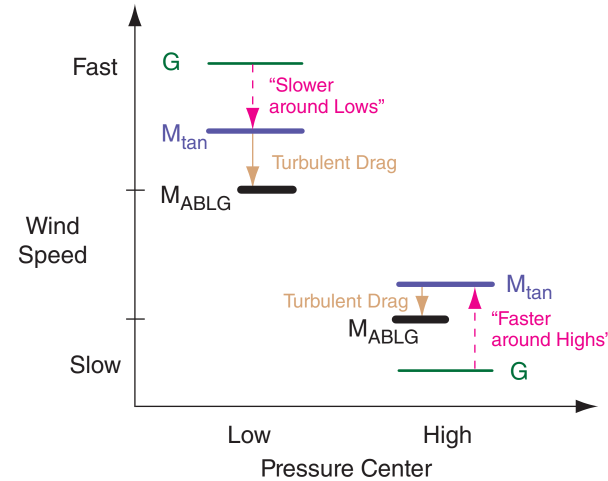


Figure 10.16
Relative wind speeds around low- & high-pressure centers. G = geostrophic wind, M_{tan} = gradient wind speed, M_{ABLG} = atmospheric-boundary-layer gradient wind speed. High-pressure centers cannot have strong pressure gradients; thus G is smaller.

Sample Application
If G = 10 m s^{-1}, find the gradient wind speed & Ro_c, given $f_c = 10^{-4}$ s^{-1} and a radius of curvature of 500 km?

Find the Answer
Given: G = 10 m s^{-1}, R = 500 km, $f_c = 10^{-4}$ s^{-1}
Find: M_{tan} = ? m s^{-1}, Ro_c = ? (dimensionless)

Use eq. (10.34a): $M_{tan} = 0.5\cdot(10^{-4}\text{s}^{-1})\cdot(500000\text{m})\cdot$

$$\left[-1+\sqrt{1+\frac{4\cdot(10m/s)}{(10^{-4}\text{s}^{-1})\cdot(500000\text{m})}}\right] = \underline{8.54 \text{ m s}^{-1}}$$

Use eq. (10.35):
$$Ro_c = \frac{(10\text{m/s})}{(10^{-4}\text{s}^{-1})\cdot(5\times10^5\text{m})} = \underline{\mathbf{0.2}}$$

Check: Physics & units are reasonable.
Exposition: The gradient wind is slower than geostrophic and is in geostrophic balance.

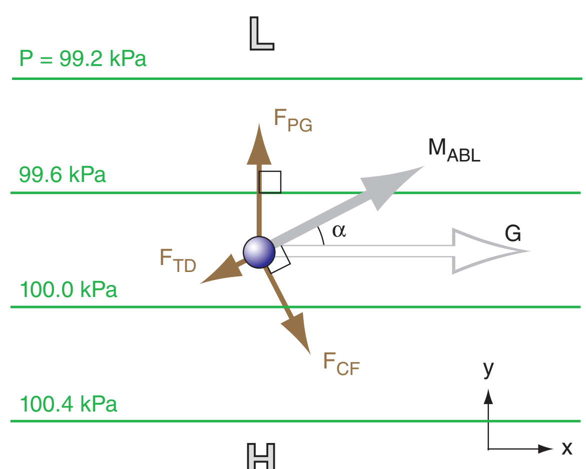


Figure 10.17
Balance of forces (brown arrows) creating an atmospheric-boundary-layer wind (M_{ABL}, solid grey arrow) that is slower than geostrophic (G, hollow grey arrow). Thin green lines are isobars. L and H are low and high-pressure centers.

Sample Application

For statically neutral conditions, find the winds in the boundary layer given: z_i = 1.5 km, U_g = 15 m s^{-1}, V_g = 0, f_c = 10^{-4} s^{-1}, and C_D = 0.003. What is the cross-isobar wind angle?

Find the Answer

Given: z_i = 1.5 km, U_g = 15 m s^{-1}, V_g = 0, f_c = 10^{-4} s^{-1}, C_D = 0.003.

Find: V_{ABL} =? m s^{-1}, U_{ABL} =? m s^{-1}, M_{ABL} =? m s^{-1}, α = ? °

First: $G = (U_g^2 + V_g^2)^{1/2}$ = 15 m s^{-1}. Now apply eq.(10.41)

$$a = \frac{0.003}{(10^{-4}\text{s}^{-1})\cdot(1500\text{m})} = 0.02 \text{ s/m}$$

Check: $a \cdot G$ = (0.02 s m^{-1})·(15 m s^{-1}) =0.3 (is < 1. Good.)

U_{ABL}=[1–0.35·(0.02s m^{-1})·(15m s^{-1})]·(15m s^{-1})≈**13.4m s^{-1}**

V_{ABL}=[1–0.5·(0.02s m^{-1})·(15m s^{-1})]·(0.02s m^{-1})·(15m s^{-1})·(15m s^{-1}) ≈ **3.8 m s^{-1}**

$$M_{ABL} = \sqrt{U_{ABL}^2 + V_{ABL}^2} = \sqrt{13.4^2 + 3.8^2} = \underline{\mathbf{13.9 \text{ m s}^{-1}}}$$

Isobars are parallel to the geostrophic wind. Thus, the cross-isobar angle is:

$\alpha = \tan^{-1}(V_{ABL}/U_{ABL}) = \tan^{-1}(3.8/13.4)$ = **15.8°** .

Check: Physics & units are reasonable.

Exposition: Drag both slows the wind (13.4 m s^{-1}) in the boundary layer below its geostrophic value (15 m s^{-1}) and turns it at a small angle (15.8°) towards low pressure. Given N. Hem. (because of the positive Coriolis parameter), the ABL wind direction is 254.2°.

Sample Application

For statically unstable conditions, find winds in the ABL given V_g = 0, U_g = 5 m s^{-1}, w_B = 50 m s^{-1}, z_i = 1.5 km, b_D = 1.83x10^{-3}, and f_c = 10^{-4} s^{-1}. What is the cross-isobar wind angle?

Find the Answer

Given: (use convective boundary layer values above)

Find: M_{ABL} =? m s^{-1}, V_{ABL} =? m s^{-1}, U_{ABL} =? m s^{-1}, α =?°

Apply eqs. (10.42):

$$c_1 = \frac{(1.83\times10^{-3})\cdot(50\text{m/s})}{(10^{-4}\text{s}^{-1})\cdot(1500\text{m})} = 0.61 \text{ (dimensionless)}$$

$c_2 = 1/[1+(0.61)^2]$ = 0.729 (dimensionless)

U_{ABL} = 0.729·[(5m s^{-1}) – 0] = **3.6 m s^{-1}**

V_{ABL} = 0.729·[0 + (0.61)·(5m s^{-1})] = **2.2 m s^{-1}**

Use eq. (10.40):

$M_{ABL} = [U^2_{ABL} + V^2_{ABL}]^{1/2}$ = **4.2 m s^{-1}**

$\alpha = \tan^{-1}(V_{ABL}/U_{ABL}) = \tan^{-1}(2.2/3.6)$ = **31.4°**

Check: Physics & units are reasonable.

Exposition: Again, drag slows the wind and causes it to cross the isobars toward low pressure. The ABL wind direction is 238.6°.

Namely, the only forces acting for this special case are pressure gradient, Coriolis, and turbulent drag (Fig. 10.17).

Replace U with U_{ABL} and V with V_{ABL} to indicate these winds are in the ABL. Eqs. (10.38) can be rearranged to solve for the ABL winds, but this solution is **implicit** (depends on itself):

$$U_{ABL} = U_g - \frac{w_T \cdot V_{ABL}}{f_c \cdot z_i} \qquad (10.39a)$$

$$V_{ABL} = V_g + \frac{w_T \cdot U_{ABL}}{f_c \cdot z_i} \qquad (10.39b)$$

where (U_g, V_g) are geostrophic wind components, f_c is Coriolis parameter, z_i is ABL depth, and w_T is the turbulent transport velocity.

You can **iterate** to solve eqs. (10.39). Namely, first you guess a value for V_{ABL} to use in the right side of the first eq. Solve eq. (10.39a) for U_{ABL} and use it in the right side of eq. (10.39b), which you can solve for V_{ABL}. Plug this back into the right side of eq. (10.39a) and repeat this procedure until the solution **converges** (stops changing very much). The magnitude of the boundary-layer wind is:

$$M_{ABL} = [U^2_{ABL} + V^2_{ABL}]^{1/2} \qquad (10.40)$$

For a statically **neutral** ABL under windy conditions, then $w_T = C_D \cdot M_{ABL}$, where C_D is the drag coefficient (eq. 10.21). For most altitudes in the neutral ABL, an approximate but explicit solution is:

$$U_{ABL} \approx (1 - 0.35 \cdot a \cdot U_g) \cdot U_g - (1 - 0.5 \cdot a \cdot V_g) \cdot a \cdot V_g \cdot G \qquad •(10.41a)$$

$$V_{ABL} \approx (1 - 0.5 \cdot a \cdot U_g) \cdot a \cdot G \cdot U_g + (1 - 0.35 \cdot a \cdot V_g) \cdot V_g \qquad •(10.41b)$$

where the parameter is $a = C_D/(f_c \cdot z_i)$, G is the geostrophic wind speed and a solution is possible only if $a \cdot G < 1$. If this condition is not met, or if no reasonable solution can be found using eqs. (10.41), then use the iterative approach described in the next section, but with the centrifugal terms set to zero. Eqs. (10.41) do not apply to the **surface layer** (bottom 5 to 10% of the neutral boundary layer).

If the ABL is statically **unstable** (e.g., sunny with slow winds), use $w_T = b_D \cdot w_B$ (see eq. 10.22). Above the surface layer there is an exact solution that is explicit:

$$U_{ABL} = c_2 \cdot [U_g - c_1 \cdot V_g] \qquad •(10.42a)$$

$$V_{ABL} = c_2 \cdot [V_g + c_1 \cdot U_g] \qquad •(10.42b)$$

where $c_1 = \dfrac{b_D \cdot w_B}{f_c \cdot z_i}$, and $c_2 = \dfrac{1}{[1 + c_1^2]}$.

The factors in c_1 are given in the "Forces" section.

In summary, both wind-shear turbulence and convective turbulence cause drag. Drag makes the ABL wind slower than geostrophic (subgeostrophic), and causes the wind to cross isobars at angle α such that it has a component pointing to low pressure.

10.5.4. ABL Gradient (ABLG) Wind

For curved isobars in the atmospheric boundary layer (ABL), there is an imbalance of the following forces: Coriolis, pressure-gradient, and drag. This imbalance is a centripetal force that makes ABL air spiral outward from highs and inward toward lows (Fig. 10.18). An example was shown in Fig. 10.1.

If we devise a centrifugal force equal in magnitude but opposite in direction to the centripetal force, then the equations of motion can be written for spiraling flow that is steady over any point on the Earth's surface (i.e., NOT following the parcel):

$$0 = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta x} + f_c \cdot V - w_T \cdot \frac{U}{z_i} + s\cdot\frac{V\cdot M}{R} \qquad (10.43a)$$

$$0 = \underbrace{-\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta y}}_{\text{pressure gradient}} \underbrace{- f_c \cdot U}_{\text{Coriolis}} \underbrace{- w_T \cdot \frac{V}{z_i}}_{\text{turbulent drag}} \underbrace{- s\cdot\frac{U\cdot M}{R}}_{\text{centrifugal}} \qquad (10.43b)$$

We can anticipate that the ABLG winds should be slower than the corresponding gradient winds, and should cross isobars toward lower pressure at some small angle α (see Fig. 10.19).

Lows are often overcast and windy, implying that the atmospheric boundary layer is statically neutral. For this situation, the transport velocity is given by:

$$w_T = C_D \cdot M = C_D \cdot \sqrt{U^2 + V^2} \qquad (10.21 \text{ again})$$

Because this parameterization is nonlinear, it increases the nonlinearity (and the difficulty to solve), eqs. (10.43).

Highs often have mostly clear skies with light winds, implying that the atmospheric boundary layer is statically unstable during sunny days, and statically stable at night. For daytime, the transport velocity is given by:

$$w_T = b_D \cdot w_B \qquad (10.22 \text{ again})$$

This parameterization for w_B is simple, and does not depend on wind speed. For statically stable conditions during fair-weather nighttime, steady state is unlikely, meaning that eqs. (10.43) do not apply.

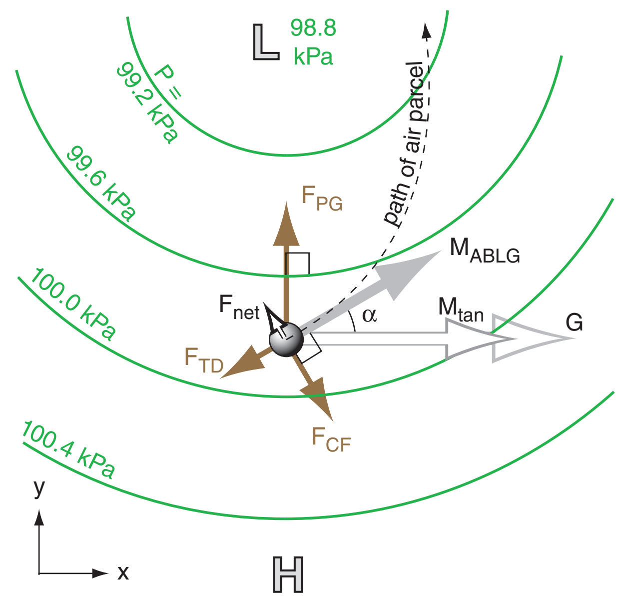


Figure 10.18
Imbalance of forces (brown arrows) yield a net centripetal force (F_{net}) that causes the atmospheric-boundary-layer gradient wind (M_{ABLG}, solid grey arrow) to be slower than both the gradient wind (M_{tan}) and geostrophic wind (G). The resulting air-parcel path crosses the isobars (green lines) at a small angle α toward low pressure.

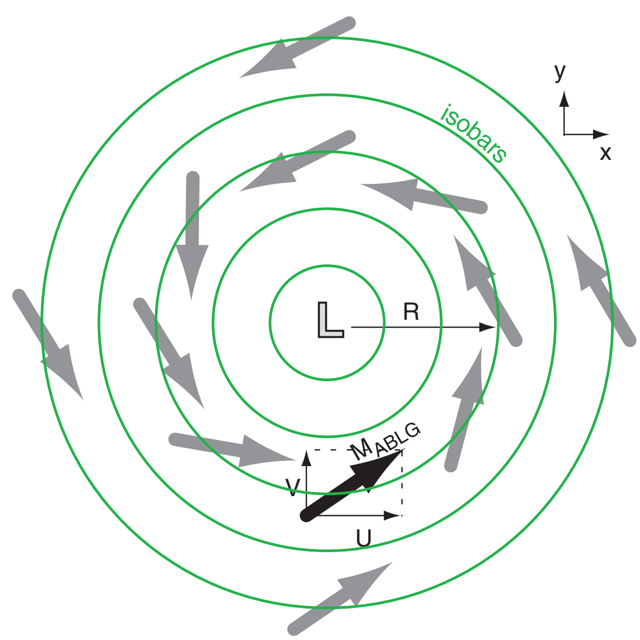


Figure 10.19
Tangential ABLG wind component (U) and radial ABLG wind component (V) for the one vector highlighted as the thick black arrow. N. Hemisphere.

Sample Application

If G = 10 m s^{-1} at R = 400 km from the center of a N. Hem. cyclone, C_D = 0.02, z_i = 1 km, and f_c = 10^{-4} s^{-1}, then find the ABLG wind speed and components.

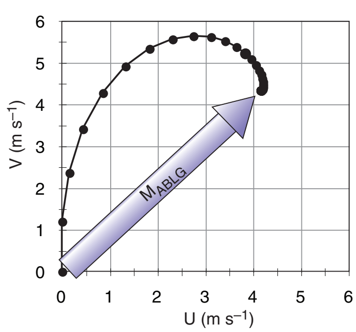
Find the Answer

Given: (see the data above)
Find: M_{BLG} = ? m s^{-1}, U_{BLG} = ? m s^{-1}, V_{BLG} = ? m s^{-1},

Use a spreadsheet to iterate (as discussed in the INFO box) eqs. (10.44) & (1.1) with a time step of Δt = 1200 s. Use U = V = 0 as a first guess.

G (m/s)=	10		z_i (km)=	1	
R (km)=	400		f_c (s^{-1})=	0.0001	
C_D =	0.02		Δt(s)=	1200	
Iteration Counter	U_{ABLG} (m s^{-1})	V_{ABLG} (m s^{-1})	M_{ABLG} (m s^{-1})	ΔU_{ABLG} (m s^{-1})	ΔV_{ABLG} (m s^{-1})
0	0.00	0.00	0.00	0.000	1.200
1	0.00	1.20	1.20	0.148	1.165
2	0.15	2.37	2.37	0.292	1.047
3	0.44	3.41	3.44	0.408	0.861
4	0.85	4.27	4.36	0.480	0.640
5	1.33	4.91	5.09	0.502	0.420
. . .					
29	4.16	4.33	6.01	-0.003	0.001
30	4.16	4.33	6.01	-0.002	0.001

The evolution of the iterative solution is plotted at right as it approaches the final answer of U_{ABLG} = **4.16 m s^{-1}**, V_{ABLG} = **4.33 m s^{-1}**, M_{ABLG}= **6.01 m s^{-1}**, where (U_{ABLG} , V_{ABLG}) are (tangential, radial) parts.



Check: Physics & units are reasonable. You should do the following "what if" experiments on the spreadsheet to check the validity. I ran experiments using a modified spreadsheet that relaxed the results using a weighted average of new and previous winds.

(a) As R approaches infinity and C_D approaches zero, then M_{ABLG} should approach the geostrophic wind G. I got U_{ABLG} = G = 10 m s^{-1}, V_{ABLG} = 0.

(b) For finite R and 0 drag, then M_{ABLG} should equal the gradient wind M_{tan}. I got U_{ABLG} = 8.28 m s^{-1}, V_{ABLG} = 0.

(c) For finite drag but infinite R, then M_{ABLG} should equal the atmospheric boundary layer wind M_{ABL}. I got U_{ABLG} = 3.91 m s^{-1}, V_{ABLG} = 4.87 m s^{-1}. Because this ABL solution is based on the full equations, it gives a better answer than eqs. (10.41).

Exposition: If you take slightly larger time steps, the solution converges faster. But if Δt is too large, the iteration method fails (i.e., blows up).

Nonlinear coupled equations (10.43) are difficult to solve analytically. However, we can rewrite the equations in a way that allows us to iterate numerically toward the answer (see the INFO box below for instructions). The trick is to not assume steady state. Namely, put the tendency terms ($\Delta U/\Delta t$, $\Delta V/\Delta t$) back in the left hand sides (LHS) of eqs. (10.43). But recall that $\Delta U/\Delta t = [U(t+\Delta t) - U]/\Delta t$, and similar for V.

For this iterative approach, first re-frame eqs. (10.43) in cylindrical coordinates, where (U, V) are the (tangential, radial) components, respectively (see Fig. 10.19). Also, use G, the geostrophic wind definition of eq. (10.28), to quantify the pressure gradient.

For a cyclone in the Northern Hemisphere (for which s = +1 from Table 10-2), the atmospheric boundary layer gradient wind eqs. (10.43) become:

$$M = (U^2 + V^2)^{1/2} \qquad \text{(1.1 again)}$$

$$U(t+\Delta t) = U + \Delta t \cdot \left[f_c \cdot V - \frac{C_D \cdot M \cdot U}{z_i} + s\frac{V \cdot M}{R} \right] \qquad \text{(10.44a)}$$

$$V(t+\Delta t) = V + \Delta t \cdot \left[f_c \cdot (G - U) - \frac{C_D \cdot M \cdot V}{z_i} - s\frac{U \cdot M}{R} \right] \qquad \text{(10.44b)}$$

where (U, V) represent (tangential, radial) parts for the wind vector south of the low center. These coupled equations are valid both night and day.

INFO • Find the Answer by Iteration

Equations (10.44) are difficult to solve analytically, but you can iterate as an alternative way to solve for the ABLG wind components. Here is the procedure.

(1) Make an initial guess for (U, V), such as (0, 0).
(2) Use these (U, V) values in the right sides of eqs. (10.44) and (1.1), and solve for the new values of [$U(t+\Delta t)$, $V(t+\Delta t)$].
(3) In preparation for the next iteration, let $U = U(t+\Delta t)$, and $V = V(t+\Delta t)$.
(4) Repeat steps 2 and 3 using the new (U, V) on the right hand sides .
(5) Keep iterating. Eventually, the [$U(t+\Delta t)$, $V(t+\Delta t)$] values stop changing (i.e., reach steady state), giving $U_{ABLG} = U(t+\Delta t)$, and $V_{ABLG} = V(t+\Delta t)$.

Because of the repeated, tedious calculations, I recommend you use a spreadsheet (see the Sample Application) or write your own computer program.

As you can see from the Sample Application, the solution spirals toward the final answer as a **damped inertial oscillation**.

For daytime fair weather conditions in anticyclones, you could derive alternatives to eqs. (10.44) that use convective parameterizations for atmospheric boundary layer drag.

Because eqs. (10.44) include the tendency terms, you can also use them for non-steady-state (time varying) flow. One such case is nighttime during fair weather (anticyclonic) conditions. Near sunset, when vigorous convective turbulence dies, the drag coefficient suddenly decreases, allowing the wind to accelerate toward its geostrophic equilibrium value. However, Coriolis force causes the winds to turn away from that steady-state value, and forces the winds into an **inertial oscillation**. See a previous INFO box titled *Approach to Geostrophy* for an example of undamped inertial oscillations.

During a portion of this oscillation the winds can become faster than geostrophic (**supergeostrophic**), leading to a low-altitude phenomenon called the **nocturnal jet**. See the Atmospheric Boundary Layer chapter for details.

10.5.5. Cyclostrophic Wind

Winds in tornadoes are about 100 m s^{-1}, and in waterspouts are about 50 m s^{-1}. As a tornado first forms and tangential winds increase, centrifugal force increases much more rapidly than Coriolis force. Centrifugal force quickly becomes the dominant force that balances pressure-gradient force (Fig. 10.20). Thus, a steady-state rotating wind is reached at much slower speeds than the gradient wind speed.

If the tangential velocity around the vortex is steady, then the steady-state force balance is:

$$0 = -\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta x} + s\cdot\frac{V\cdot M}{R} \qquad \text{(10.45a)}$$

$$0 = -\underbrace{\frac{1}{\rho}\cdot\frac{\Delta P}{\Delta y}}_{\text{pressure gradient}} - s\cdot\underbrace{\frac{U\cdot M}{R}}_{\text{centrifugal}} \qquad \text{(10.45b)}$$

You can use cylindrical coordinates to simplify solution for the cyclostrophic (tangential) winds M_{cs} around the vortex. The result is:

$$M_{cs} = \sqrt{\frac{R}{\rho}\cdot\frac{\Delta P}{\Delta R}} \qquad \text{(10.46)}$$

where the velocity M_{cs} is at distance R from the vortex center, and the radial pressure gradient in the vortex is $\Delta P/\Delta R$.

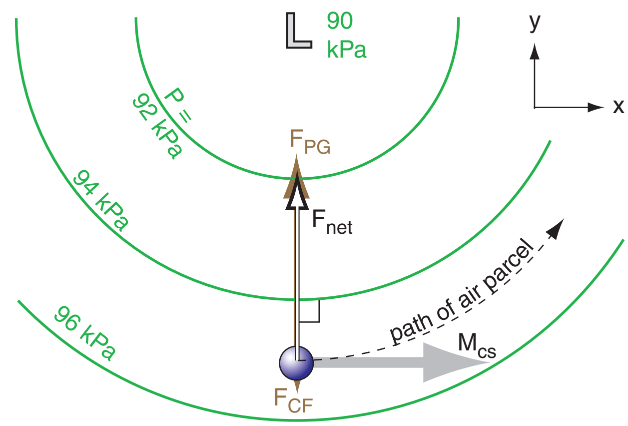


Figure 10.20
Around tornadoes, pressure gradient force F_{PG} is so strong that it greatly exceeds all other forces such as Coriolis force F_{CF}. The net force (F_{net}) pulls the air around the tight circle at the cyclostrophic wind speed (M_{cs}).

Sample Application
A 10 m radius waterspout has a tangential velocity of 45 m s^{-1}. What is the radial pressure gradient?

Find the Answer
Given: M_{cs} = 45 m s^{-1}, R = 10 m.
Find: $\Delta P/\Delta R$ = ? kPa m^{-1}.

Assume cyclostrophic wind, and ρ = 1 kg m^{-3}.
Rearrange eq. (10.46):

$$\frac{\Delta P}{\Delta R} = \frac{\rho}{R}\cdot M_{cs}^2 = \frac{(1\text{kg/m}^3)\cdot(45\text{m/s})^2}{10\text{m}}$$

$\Delta P/\Delta R$ = 202.5 kg·m^{-1}·s^{-2} / m = **0.2 kPa m^{-1}**.

Check: Physics & units are reasonable.
Exposition: This is 2 kPa across the 10 m waterspout radius, which is 1000 times greater than typical synoptic-scale pressure gradients on weather maps.

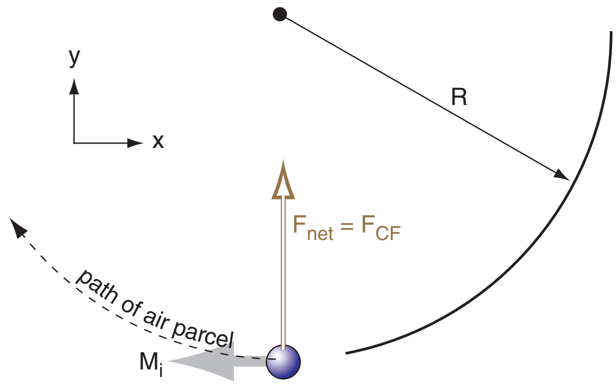


Figure 10.21
Coriolis force (F_{CF}, thick brown arrow, behind the white arrow) on an air parcel (blue ball), creating an anticyclonic inertial wind M_i, grey arrow). R is radius of curvature. White arrow is net force F_{net}.

Sample Application
For an inertial ocean current of 5 m s^{-1}, find the radius of curvature and time period to complete one circuit. Assume a latitude where $f_c = 10^{-4}$ s^{-1}.

Find the Answer
Given: $M_i = 5$ m s^{-1}, $f_c = 10^{-4}$ s^{-1}.
Find: R = ? km, *Period* = ? h

Use eq. (10.48): $R = -(5 \text{ m s}^{-1}) / (10^{-4} \text{ s}^{-1})$ = **−50 km**
Use *Period* $= 2\pi/f_c$ = 62832 s = **17.45 h**

Check: Units & magnitudes are reasonable.
Exposition: The tracks of drifting buoys in the ocean are often **cycloidal**, which is the superposition of a circular inertial oscillation and a mean current that gradually translates (moves) the whole circle.

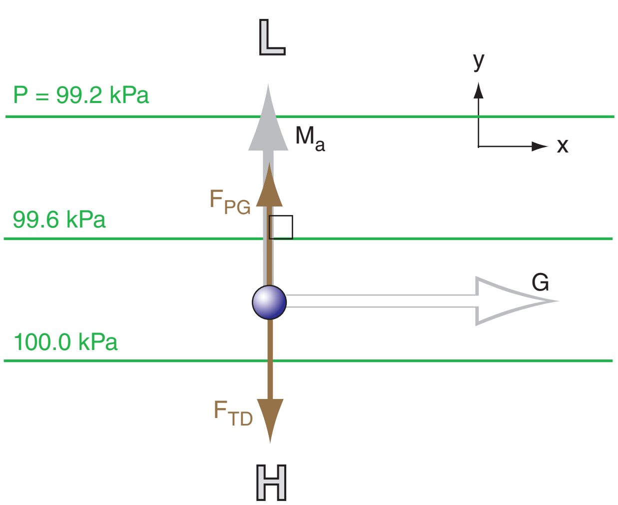


Figure 10.22
Balance of forces (F, brown arrows) that create the antitriptic wind M_a (grey arrow). G is the theoretical geostrophic wind. F_{TD} is turbulent drag, and F_{PG} is pressure-gradient force.

Recall from the Gradient Wind section that anticyclones cannot have strong pressure gradients, hence winds around highs are too slow to be cyclostrophic. Around cyclones (lows), cyclostrophic winds can turn either counterclockwise or clockwise in either hemisphere, because Coriolis force is not a factor.

10.5.6. Inertial Wind

Steady-state **inertial motion** results from a balance of Coriolis and centrifugal forces in the absence of any pressure gradient:

$$0 = f_c \cdot M_i + \frac{M_i^2}{R} \tag{10.47}$$

where M_i is inertial wind speed, f_c is the Coriolis parameter, and R is the radius of curvature. Since both of these forces depend on wind speed, the inertial wind cannot start itself from zero. It can occur only after some other force first causes the wind to blow, and then that other force disappears.

The inertial wind coasts around a circular path of radius R,

$$R = -\frac{M_i}{f_c} \tag{10.48}$$

where the negative sign implies anticyclonic rotation (Fig. 10.21). The time period needed for this **inertial oscillation** to complete one circuit is *Period* $= 2\pi/f_c$, which is half of a **pendulum day** (see *Approach to Geostrophy* INFO Box earlier in this chapter).

Although rarely observed in the atmosphere, inertial oscillations are frequently observed in the ocean. This can occur where wind stress on the ocean surface creates an ocean current, and then after the wind dies the current coasts in an inertial oscillation.

10.5.7. Antitriptic Wind

A steady-state antitriptic wind M_a could result from a balance of pressure-gradient force and turbulent drag:

$$0 = -\frac{1}{\rho} \cdot \frac{\Delta P}{\Delta d} - w_T \cdot \frac{M_a}{z_i} \tag{10.49}$$

where ΔP is the pressure change across a distance Δd perpendicular to the isobars, w_T is the turbulent transport velocity, and z_i is the atmospheric boundary-layer depth.

This theoretical wind blows perpendicular to the isobars (Fig. 10.22), directly from high to low pressure:

$$M_a = \frac{z_i \cdot f_c \cdot G}{w_T} \qquad (10.50)$$

For free-convective boundary layers, $w_T = b_D \cdot w_B$ is not a function of wind speed, so M_a is proportional to G. However, for windy forced-convection boundary layers, $w_T = C_D \cdot M_a$, so solving for M_a shows it to be proportional to the square root of G.

This wind would be found in the atmospheric boundary layer, and would occur as an along-valley component of "long gap" winds (see the Regional Winds chapter). It is also sometimes thought to be relevant for thunderstorm cold-air outflow and for steady sea breezes. However, in most other situations, Coriolis force should not be neglected; thus, the atmospheric boundary-layer wind and BL Gradient winds are much better representations of nature than the antitriptic wind.

10.5.8. Summary of Horizontal Winds

Table 10-5 summarizes the idealized horizontal winds that were discussed earlier in this chapter.

On real weather maps such as Fig. 10.23, isobars or height contours have complex shapes. In some regions the height contours are straight (suggesting that actual winds should nearly equal geostrophic or boundary-layer winds), while in other regions the height contours are curved (suggesting gradient or boundary-layer gradient winds). Also, as air parcels move between straight and curved regions, they are sometimes not quite in equilibrium. Nonetheless, when studying weather maps you can quickly estimate the winds using the summary table.

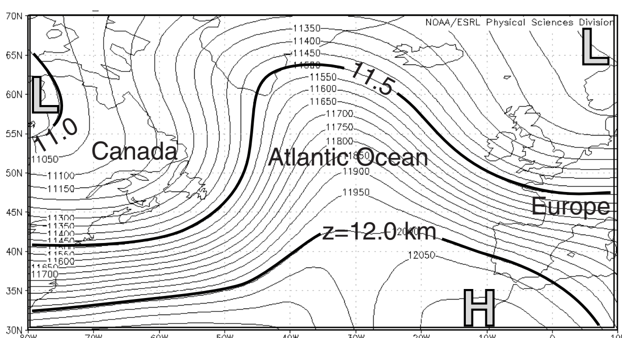


Figure 10.23
One-day average geopotential heights z (thick lines in km, thin lines in m) on the 20 kPa isobaric surface for 5 Feb 2013. Close spacing (tight packing) of the height contours indicate faster winds. This upper-level chart is for the same day and location (Atlantic Ocean) as the mean-sea-level pressure chart in Fig. 10.15. [Courtesy of NOAA/NCEP Earth System Research Lab. http://www.esrl.noaa.gov/psd/data/gridded/data.ncep.reanalysis.html]

Sample Application

In a 1 km thick convective boundary layer at a location where $f_c = 10^{-4}$ s^{-1}, the geostrophic wind is 5 m s^{-1}. The turbulent transport velocity is 0.02 m s^{-1}. Find the antitriptic wind speed.

Find the Answer

Given: $G = 5$ m s^{-1}, $z_i = 1000$ m, $f_c = 10^{-4}$ s^{-1},
$w_T = 0.02$ m s^{-1}
Find: $M_a = ?$ m s^{-1}

Use eq. (10.50):
$M_a = (1000\text{m})\cdot(10^{-4}\text{ s}^{-1})\cdot(5\text{m s}^{-1}) / (0.02\text{ m s}^{-1})$
$= \underline{\mathbf{25\ m\ s^{-1}}}$

Check: Magnitude is too large. Units reasonable.
Exposition: Eq. (10.50) can give winds of $M_a > G$ for many convective conditions, for which case Coriolis force would be expected to be large enough that it should not be neglected. Thus, antitriptic winds are unphysical. However, for forced-convective boundary layers where drag is proportional to wind speed squared, reasonable solutions are possible.

Table 10-5. Summary of horizontal winds**.

Item	Name of Wind	Forces	Direction	Magnitude	Where Observed
1	**geostrophic**	pressure-gradient, Coriolis	parallel to straight isobars with Low pressure to the wind's left*	faster where isobars are closer together. $G = \left\lvert \frac{g}{f_c} \cdot \frac{\Delta z}{\Delta d} \right\rvert$	aloft in regions where isobars are nearly straight
2	**gradient**	pressure-gradient, Coriolis, centrifugal	similar to geostrophic wind, but following curved isobars. Clockwise* around Highs, counterclockwise* around Lows.	slower than geostrophic around Lows, faster than geostrophic around Highs	aloft in regions where isobars are curved
3	**atmospheric boundary layer**	pressure-gradient, Coriolis, drag	similar to geostrophic wind, but crosses isobars at small angle toward Low pressure	slower than geostrophic (i.e., **subgeostrophic**)	near the ground in regions where isobars are nearly straight
4	**atmospheric boundary-layer gradient**	pressure-gradient, Coriolis, drag, centrifugal	similar to gradient wind, but crosses isobars at small angle toward Low pressure	slower than gradient wind speed	near the ground in regions where isobars are curved
5	**cyclostrophic**	pressure-gradient, centrifugal	either clockwise or counterclockwise around strong vortices of small diameter	stronger for lower pressure in the vortex center	tornadoes, waterspouts (& sometimes in the eye-wall of hurricanes)
6	**inertial**	Coriolis, centrifugal	anticyclonic circular rotation	coasts at constant speed equal to its initial speed	ocean-surface currents

** For Northern Hemisphere. Direction is opposite in Southern Hemisphere. ** Antitriptic winds are unphysical; not listed here.*

10.6. HORIZONTAL MOTION

10.6.1. Equations of Motion — Again

The geostrophic wind can be used as a surrogate for the pressure-gradient force, based on the definitions in eqs. (10.26). Thus, the **equations of horizontal motion** (10.23) become:

•(10.51a)

$$\frac{\Delta U}{\Delta t} = -U\frac{\Delta U}{\Delta x} - V\frac{\Delta U}{\Delta y} - W\frac{\Delta U}{\Delta z} + f_c\cdot\left(V - V_g\right) - w_T\cdot\frac{U}{z_i}$$

•(10.51b)

$$\underbrace{\frac{\Delta V}{\Delta t}}_{tendency} = \underbrace{-U\frac{\Delta V}{\Delta x} - V\frac{\Delta V}{\Delta y} - W\frac{\Delta V}{\Delta z}}_{advection} \underbrace{- f_c\cdot}_{Coriolis}\underbrace{\left(U - U_g\right)}_{pressure\ gradient} \underbrace{- w_T\cdot\frac{V}{z_i}}_{turbulent\ drag}$$

For winds turning around a circle, you can add a term for centrifugal force, which is an artifice to account for the continual changing of wind direction caused by an imbalance of the other forces (where the imbalance is the centripetal force).

The difference between the actual and geostrophic winds is the **ageostrophic wind** (U_{ag}, V_{ag}). The term in eqs. (10.51) containing these differences indicates the **geostrophic departure**.

$$U_{ag} = U - U_g \qquad •(10.52a)$$

$$V_{ag} = V - V_g \qquad •(10.52b)$$

10.6.2. Scales of Horizontal Motion

A wide range of horizontal scales of motion (Table 10-6) are superimposed in the atmosphere: from large global-scale circulations through extra-tropical cyclones, thunderstorms, and down to swirls of turbulence.

The troposphere is roughly 10 km thick, and this constrains the vertical scale of most weather phenomena. Thus, phenomena of large horizontal scale will have a constrained vertical scale, causing them to be similar to a pancake. However, phenomena with smaller horizontal scale can have aspect ratios (width/height) of about one; namely, their characteristics are **isotropic**.

Larger-scale meteorological phenomena tend to exist for longer durations than smaller-scale ones. Fig. 10.24 shows that time scales τ and horizontal length scales λ of many meteorological phenomena nearly follow a straight line on a log-log plot. This implies that

$$\tau/\tau_o = (\lambda/\lambda_o)^b \qquad (10.53)$$

where $\tau_o \approx 10^{-3}$ h, $\lambda_o \approx 10^{-3}$ km, and $b \approx 7/8$.

In the next several chapters, we cover weather phenomena from largest to smallest horiz. scales:

- Chapter 11 General Circulation (planetary)
- Chapter 12 Fronts & Airmasses (synoptic)
- Chapter 13 Extratropical Cyclones (synoptic)
- Chapter 14 Thunderstorm Fundam. (meso β)
- Chapter 15 Thunderstorm Hazards (meso γ)
- Chapter 16 Tropical Cyclones (meso α & β)
- Chapter 17 Regional Winds (meso β & γ)
- Chapter 18 Atm. Boundary Layers (microscale)

Although hurricanes are larger than thunderstorms, we cover thunderstorms first because they are the building blocks of hurricanes. Similarly, midlatitude cyclones often contain fronts, so fronts are covered before extratropical cyclones.

Table 10-6. Horizontal scales of motion in the troposphere.

Horizontal Size	Scale: Designation	Scale: Name	
40,000 km – 4,000 km	macro α	planetary scale	
4,000 km – 700 km	macro β	synoptic scale*	
700 km – 300 km	meso α	mesoscale**	
300 km – 30 km	meso β	mesoscale**	
30 km – 3 km	meso γ	mesoscale**	
3 km – 300 m	micro α	microscale***	boundary-layer turbulence
300 m – 30 m	micro β	microscale***	surface-layer turbulence
30 m – 3 m	micro γ	microscale***	inertial subrange turbulence
3 m – 300 mm	micro δ	microscale***	inertial subrange turbulence
300 mm – 3 mm	micro δ	microscale***	fine-scale turbulence
3 mm – 0.3 μm	viscous	dissipation subrange	
0.3 μm – 0.003 μm	molecular	mean-free path between molec.	
0.003 μm – 0	molecular	molecule sizes	

Note: Disagreement among different organizations.
*Synoptic: AMS: 400 - 4000 km; WMO: 1000 - 2500 km.
**Mesoscale: AMS: 3 - 400 km; WMO: 3 - 50 km.
***Microscale: AMS: 0 - 2 km; WMO: 3 cm - 3 km.
where AMS = American Meteorological Society,
and WMO = World Meteorological Organization.

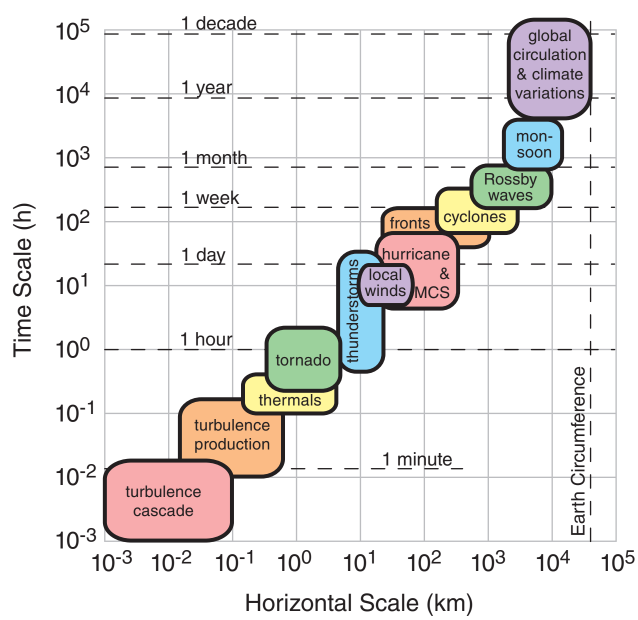


Figure 10.24
Typical time and spatial scales of meteorological phenomena. MCS = Mesoscale Convective System (see the thunderstorm chapter).

10.7. VERTICAL FORCES AND MOTION

Forces acting in the vertical can cause or change vertical velocities, according to Newton's Second Law. In an Eulerian framework, the **vertical component of the equations of motion** is:

$$\underbrace{\frac{\Delta W}{\Delta t}}_{tendency} = \underbrace{-U\frac{\Delta W}{\Delta x} - V\frac{\Delta W}{\Delta y} - W\frac{\Delta W}{\Delta z}}_{advection} \underbrace{-\frac{1}{\rho}\frac{\Delta P}{\Delta z}}_{pressure\ gradient} \underbrace{-|g|}_{gravity} \underbrace{-\frac{F_{z\,TD}}{m}}_{turb.\ drag} \qquad (10.54)$$

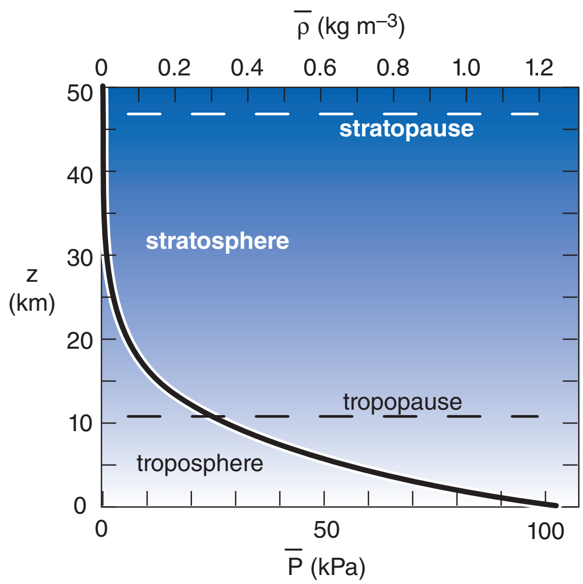


Figure 10.25
Background state, showing change of mean atmospheric pressure $\overline{P}$ and mean density $\overline{\rho}$ with height z, based on a standard atmosphere from Chapter 1.

where the vertical acceleration given in the left side of the equation is determined by the sum of all forces/mass acting in the vertical, as given on the right. For Cartesian directions (x, y, z) the velocity components are (U, V, W). Also in this equation are air density (ρ), pressure (P), vertical turbulent-drag force ($F_{z\ TD}$), mass (m), and time (t). Magnitude of gravitational acceleration is $|g|$ = 9.8 m·s^{-2}. Coriolis force is negligible in the vertical (see the INFO box on *Coriolis Force in 3-D*, earlier in this chapter), and is not included in the equation above.

Recall from Chapter 1 that our atmosphere has an extremely large pressure gradient in the vertical, which is almost completely balanced by gravity (Fig. 10.25). Also, there is a large density gradient in the vertical. We can define these large terms as a **mean background state** or a **reference state** of the atmosphere. Use the overbar over variables to indicate their average background state. Define this background state such that it is exactly in **hydrostatic balance** (see Chapter 1):

$$\frac{\Delta \overline{P}}{\Delta z} = -\overline{\rho}\cdot|g| \tag{10.55}$$

However, small <u>deviations</u> in density and pressure from the background state can drive important **non-hydrostatic** vertical motions, such as in thermals and thunderstorms. To discern these effects, we must first remove the background state from the full vertical equation of motion. From eq. (10.54), the gravity and pressure-gradient terms are:

$$\frac{1}{\rho}\left[-\frac{\Delta P}{\Delta z} - \rho|g|\right] \tag{10.56}$$

But total density ρ can be divided into background ($\overline{\rho}$) and <u>deviation</u> (ρ') components: $\rho = \overline{\rho} + \rho'$. Do the same for pressure: $P = \overline{P} + P'$. Thus, eq. (10.56) can be expanded as:

$$\frac{1}{(\overline{\rho}+\rho')}\left[-\frac{\Delta \overline{P}}{\Delta z} - \frac{\Delta P'}{\Delta z} - \overline{\rho}|g| - \rho'|g|\right] \tag{10.57}$$

The first and third terms in square brackets in eq. (10.57) cancel out, due to hydrostatic balance (eq. 10.55) of the background state.

In the atmosphere, density perturbations (ρ') are usually much smaller than mean density. Thus density perturbations can be neglected everywhere except in the gravity term, where $\rho'|g|/(\overline{\rho}+\rho') \approx (\rho'/\overline{\rho})\cdot|g|$. This is called the **Boussinesq approximation**.

Recall from the chapters 1 and 5 that you can use virtual temperature (T_v) with the ideal gas law in place of air density (but changing the sign because low virtual temperatures imply high densities):

Sample Application

Suppose your neighborhood has a background environmental temperature of 20°C, but at your particular location the temperature is 26°C with a 4 m s^{-1} west wind and no vertical velocity. Just 3 km west is an 5 m s^{-1} updraft. Find the vertical acceleration.

Find the Answer

Given: T_e = 273+20 = 293 K, $\Delta\theta$ = 6°C, U= 4 m s^{-1}
$\Delta W/\Delta x$= (5 m s^{-1} – 0) / (–3,000m – 0)
Find: $\Delta W/\Delta t$ = ? m·s^{-2}

Assume: Because W = 0, there is zero drag. Because the air is dry: $T_v = T$. Given no V info, assume zero.

Apply eq. (10.59): $\dfrac{\Delta W}{\Delta t} = -U\dfrac{\Delta W}{\Delta x} + \dfrac{\theta_p - \theta_e}{\overline{T}_e}\cdot|g|$

$\Delta W/\Delta t$ = –(4 m s^{-1})·(–5m·s^{-1}/3,000m)+(6/293)·(9.8m·s^{-2})
= 0.0067 + 0.20 = **<u>0.21 m·s</u>**$^{-2}$

Check: Physics & units are reasonable.
Exposition: Buoyancy dominated over advection for this example. Although drag was zero initially because of zero initial vertical velocity, we must include the drag term once the updraft forms.

$$-\frac{\rho'}{\bar{\rho}}\cdot|g| \; = \; \frac{\theta'_v}{\bar{T}_v}\cdot|g| \; = \; \frac{\theta_{v\,p}-\theta_{v\,e}}{\bar{T}_{v\,e}}\cdot|g| \; = \; g' \quad (10.58)$$

where subscripts p & e indicate the air parcel and the environment surrounding the parcel, and where g' is called the **reduced gravity**. The virtual potential temperature θ_v can be in either Celsius or Kelvin, but units of Kelvin must be used for T_v and T_{ve}.

Combining eqs. (10.54) & (10.58) yields:

$$\frac{\Delta W}{\Delta t} = -U\frac{\Delta W}{\Delta x} - V\frac{\Delta W}{\Delta y} - W\frac{\Delta W}{\Delta z} \qquad \bullet(10.59)$$

$$-\frac{1}{\bar{\rho}}\frac{\Delta P'}{\Delta z} + \frac{\theta_{vp}-\theta_{ve}}{\bar{T}_{ve}}\cdot|g| - \frac{F_{z\,TD}}{m}$$

Terms from this equation will be used in the Regional Winds chapter and in the Thunderstorm chapters to explain strong vertical velocities.

When an air parcel rises or sinks it experiences resistance (turbulent drag, $F_{z\,TD}$) per unit mass m as it tries to move through the surrounding air. This is a completely different effect than air drag against the Earth's surface, and is not described by the same drag equations. The nature of $F_{z\,TD}$ is considered in the chapter on Air Pollution Dispersion, as it affects the rise of smoke-stack plumes. $F_{z\,TD} = 0$ if the air parcel and environment move at the same speed.

10.8. CONSERVATION OF AIR MASS

Due to random jostling, air molecules tend to distribute themselves uniformly within any volume. Namely, the air tends to maintain its **continuity**. Any additional air molecules entering the volume that are not balanced by air molecules leaving (Fig. 10.26) will cause the air density (ρ, mass of air molecules in the volume) to increase, as described below by the **continuity equation**.

10.8.1. Continuity Equation

For a fixed Eulerian volume, the **mass budget equation** (i.e., the **continuity equation**) is:

$$\frac{\Delta\rho}{\Delta t} = \left\{-U\frac{\Delta\rho}{\Delta x} - V\frac{\Delta\rho}{\Delta y} - W\frac{\Delta\rho}{\Delta z}\right\} - \rho\left[\frac{\Delta U}{\Delta x} + \frac{\Delta V}{\Delta y} + \frac{\Delta W}{\Delta z}\right] \qquad (10.60)$$

The terms in curly braces { } describe advection. With a bit of calculus one can rewrite this equation as:

INFO • Eötvös Effect

When you move along a path at constant distance R above Earth's center, gravitational acceleration appears to change slightly due to your motion. The measured gravity $|g_{obs}| = |g| - a_r$, where:

$$a_r = 2\cdot\Omega\cdot\cos(\phi)\cdot U \; + \; (U^2 + V^2)/R$$

The first term is the vertical component of Coriolis force (eq. 10.17e in the INFO box on p.297), and the last term is centrifugal force as you follow the curvature of the Earth. Thus, you feel lighter traveling east and heavier traveling west. This is the **Eötvös effect**.

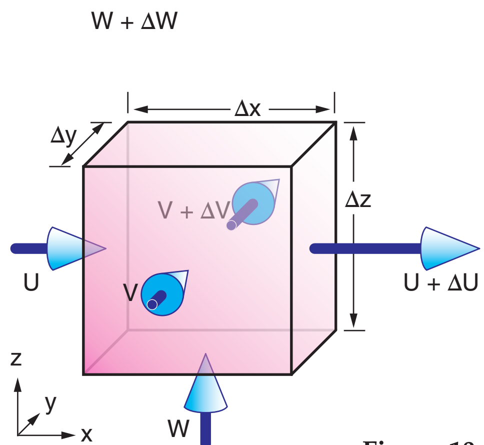


Figure 10.26
Air-mass flows to and from a fixed Eulerian volume.

Sample Application

Hurricane-force winds of 60 m s^{-1} blow into an north-facing entrance of a 20 m long pedestrian tunnel. The door at the other end of the tunnel is closed. The initial air density in the tunnel is 1.2 kg m^{-3}. Find the rate of air density increase in the tunnel.

Find the Answer

Given: $V_{N.\,entrance} = -60$ m s^{-1}, $V_{S.\,entrance} = 0$ m s^{-1}, ρ = 1.2 kg m^{-3}, $\Delta y = 20$ m,

Find: $\Delta\rho/\Delta t$ = ? kg·m^3·s^{-1} initially.

Use eq. (10.60), with $U = W = 0$ because the other walls, roof, and floor prevent winds in those directions:

$$\frac{\Delta\rho}{\Delta t} = -\rho\frac{V_{N.entr.} - V_{S.entr.}}{\Delta y_{tunnel}} = -\left(1.2\,\frac{\text{kg}}{\text{m}^3}\right)\frac{(-60-0)\text{m/s}}{(20)\text{m}}$$

$\Delta\rho/\Delta t$ = <u>+3.6</u> kg·m^{-3}·s^{-1} .

Check: Physics & units are reasonable.

Exposition: As air density increases, so will air pressure. This pressure might be sufficient to blow open the other door at the south end of the pedestrian tunnel, allowing the density to decrease as air escapes.

$$\frac{\Delta\rho}{\Delta t} = -\left[\frac{\Delta(\rho U)}{\Delta x} + \frac{\Delta(\rho V)}{\Delta y} + \frac{\Delta(\rho W)}{\Delta z}\right] \qquad (10.61)$$

where U, V, and W are the wind components in the x, y, and z directions, respectively, and t is time.

When you calculate wind gradients, be sure to take the wind and space differences in the same direction. For example: $\Delta U/\Delta x = (U_2–U_1)/(x_2–x_1)$.

10.8.2. Incompressible Idealization

Mean air density changes markedly with altitude, as was sketched in Fig. 10.25. However, at any one altitude the density changes only slightly due to local changes in humidity and temperature. For non-tornadic, non-thunderstorm conditions where Fig. 10.25 is valid, we can make a reasonable simplifying idealization that density is constant ($\Delta\rho \approx 0$) at any one altitude. Namely, air behaves as if it is **incompressible**.

If we make this idealization, then the advection terms of eq. (10.60) are zero, and the time-tendency term is zero. The net result is **volume conservation**, where volume outflow equals volume inflow:

$$\frac{\Delta U}{\Delta x} + \frac{\Delta V}{\Delta y} + \frac{\Delta W}{\Delta z} = 0 \qquad \bullet(10.62)$$

Fig. 10.26 illustrates such **incompressible continuity**. Can you detect an error in this figure? It shows more air leaving the volume in each coordinate direction than is entering — impossible for incompressible flow. A correct figure would have changed arrow lengths, to indicate net <u>in</u>flow in one or two directions, balanced by net <u>out</u>flow in the other direction(s).

As will be explained in the last section of this chapter, **divergence** is where more air leaves a volume than enters (corresponding to positive terms in eq. 10.62). **Convergence** is where more air enters than leaves (corresponding to negative terms in eq. 10.62). Thus, volume (mass) conservation of incompressible flow requires one or two terms in eq. (10.62) to be negative (i.e., convergence), and the remaining term(s) to be positive (i.e., divergence) so that their sum equals zero.

Horizontal divergence (D) is defined as

$$D = \frac{\Delta U}{\Delta x} + \frac{\Delta V}{\Delta y} \qquad (10.63)$$

Negative values of D correspond to convergence. Plugging this definition into eq. (10.62) shows that vertical velocities increase with height where there is horizontal convergence:

$$\frac{\Delta W}{\Delta z} = -D \qquad (10.64)$$

10.8.3. Boundary-Layer Pumping

Consider an extratropical cyclone, where the boundary-layer gradient wind spirals in toward the low-pressure center. Those spiraling winds consist of a tangential component following the isobars as they encircle the low center, and a radial component having inflow velocity V_{in} (Fig. 10.27).

But volume inflow ($2\pi R{\cdot}\Delta z \cdot V_{in}$) through the sides of the cylindrical volume of radius R and height Δz must be balanced by net volume outflow ($\pi R^2 \cdot \Delta W/\Delta z$) through the top and bottom. Equating these incompressible flows gives:

$$\frac{2 \cdot V_{in}}{R} = \frac{\Delta W}{\Delta z} \qquad \bullet(10.65a)$$

Thus, for horizontal inflow everywhere (positive V_{in}), one finds that ΔW must also be positive.

If a cylinder of air is at the ground where $W = 0$ at the cylinder bottom, then W at the cylinder top is:

$$W = (2 \cdot V_{in} \cdot \Delta z) \ / \ R \qquad (10.65b)$$

Namely, extratropical cyclones have rising air, which causes clouds and rain due to adiabatic cooling. This forcing of a broad updraft region by horizontal-wind drag around a cyclone is known as **boundary-layer pumping** or **Ekman pumping**.

For atmospheric boundary-layer gradient (ABLG) winds around anticyclones (highs), the opposite occurs: horizontal outflow and a broad region of descending air (**subsidence**). The subsidence causes adiabatic warming, which evaporates any clouds and creates fair weather.

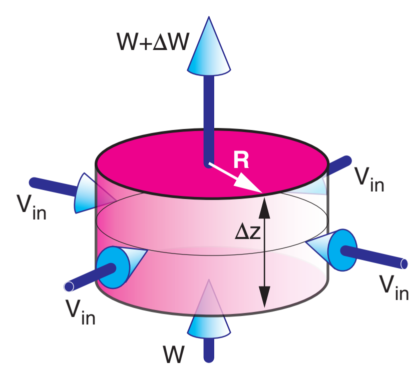


Figure 10.27
Volume conservation for an idealized cylindrical extratropical cyclone.

Recall from the ABLG wind section that an analytical solution could not be found for V_{ABLG} (which is the needed V_{in} for eq. 10.65). Instead, we can approximate $V_{in} \approx V_{ABL}$ for which an analytical solution exists. But V_{ABL} is always larger than V_{ABLG} for flow around cyclones, so we must be aware that our analytical answer will always give winds that are slightly faster than occur around lows in nature.

To solve for V_{ABL}, we need to make an assumption about the static stability of the atmospheric boundary layer. Because cyclones generally have overcast skies and strong winds, we can safely assume neutral stability. In this case, eq. (10.41b) gives the cross-isobaric inflow velocity.

Use V_{ABL} for V_{in} in eq. (10.65b) and solve for W (which we will call W_{ABL} — the vertical velocity at the atmospheric boundary-layer top, as sketched in Fig. 10.28):

$$W_{ABL} = \frac{2 \cdot b \cdot C_D}{f_c} \cdot \frac{G^2}{R} \qquad \bullet(10.66)$$

with geostrophic wind G, radius of curvature R, Coriolis parameter f_c, and drag coefficient C_D for statically neutral boundary conditions. For flow over land, $C_D \approx 0.005$.

Eq. (10.41b) can be used to find $b = \{ 1 - 0.5 \cdot [C_D \cdot G/(f_c \cdot z_i)] \}$ for an atmospheric boundary layer of thickness z_i. If you don't know the actual **atmospheric boundary-layer depth**, then a crude approximation for cyclones (not valid for anticyclones) is :

$$z_i \approx \frac{G}{N_{BV}} \qquad (10.67)$$

In this approximation, you must use a Brunt-Väisälä frequency N_{BV} that is valid for the statically stable air in the troposphere <u>above</u> the top of the statically neutral atmospheric boundary layer. For this special approximation: $b = \{ 1 - 0.5 \cdot [C_D \cdot N_{BV}/f_c] \}$. A required condition for a physically realistic solution is $[C_D \cdot N_{BV}/f_c] < 1$.

You can interpret eq. (10.66) as follows. Stronger pressure gradients (which cause larger geostrophic wind G), larger drag coefficients, and smaller radii of curvature cause greater atmospheric boundary-layer pumping W_{ABL}.

Although the equations above allow a complete approximate solution, we can rewrite them in terms of a **geostrophic relative vorticity**:

$$\zeta_g = \frac{2 \cdot G}{R} \qquad (10.68)$$

which indicates air rotation. Vorticity is introduced later in this chapter, and is covered in greater detail in the General Circulation chapter.

Sample Application

Atmospheric boundary-layer gradient winds have an inflow component of 3 m s^{-1} at radius 500 km from the center of a midlatitude cyclone. What is the updraft speed?

Find the Answer

Given: R = 500 km, V_{in} = 3 m s^{-1}. Let $\Delta z = z_i$ = 1 km

Find: W = ? m s^{-1}

Apply eq. (10.65b):

$$W = 2 \cdot V_{in} \cdot \frac{\Delta z}{R} = 2 \cdot (3\text{m/s}) \cdot \frac{1\text{km}}{500\text{km}}$$

$$= \underline{\mathbf{0.012 \text{ m s}^{-1}}}$$

Check: Physics & units are reasonable.

Exposition: This updraft speed of 1.2 cm/s, while slow, can cause significant lifting over many hours. Nonetheless, this updraft is slow enough to enable us to approximate the atmosphere as being hydrostatic.

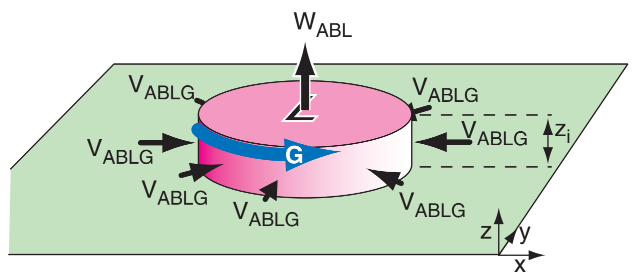


Figure 10.28
Application of atmospheric boundary-layer gradient winds V_{ABLG} to estimate the vertical velocity W_{ABL} due to atmospheric boundary-layer pumping around a low-pressure center (L). G = geostrophic wind.

Sample Application

At 500 km from the center of a midlatitude cyclone at latitude where f_c = 0.0001 s^{-1}, the pressure gradient can drive a 15 m s^{-1} geostrophic wind. Assume a standard atmosphere static stability above the top of the atmospheric boundary layer (ABL), and a drag coefficient of 0.004 at the bottom. Find the Ekman pumping updraft speed out of the atmospheric boundary-layer top. Also, what are the geostrophic relative vorticity, the depth of the ABL, and the internal Rossby deformation radius?

Find the Answer

Given: f_c = 0.0001 s^{-1}, R = 5x10^5 m, G = 15 m s^{-1}, C_D = 0.004,

Find: W_{ABL} = ? m s^{-1}, ζ_g = ? s^{-1}, z_i = ? m, λ_R = ? km

For depth of the troposphere, assume z_T = 11 km.

First, to get the Brunt-Väisälä frequency, use the Standard-Atmosphere temperatures (see Chapter 1) at the top and bottom of the troposphere to estimate the average temperature and vertical temperature gradient:

T_{avg} = 0.5·(–56.5 + 15.0) = –20.8°C = 252K

ΔT = (–56.5 – 15.0)°C = –71.5°C across Δz = 11 km

Use these in eq (5.4a):

$$N_{BV} = \sqrt{\frac{9.8\text{m/s}}{252\text{K}}\left[\frac{-71.5\text{K}}{11000\text{m}} + 0.0098\frac{\text{K}}{\text{m}}\right]} = 0.0113 \text{ s}^{-1}$$

Apply eq. (10.67):

$z_i \approx G/N_{BV}$ = (15 m s^{-1})/(0.0113 s^{-1}) = **1327 m**

Apply eq. (10.68):

$$\zeta_g = \frac{2\cdot(15\text{m/s})}{5\times10^5\text{m}} = \mathbf{6x10^{-5}}\ \text{s}^{-1}$$

Apply eq. (10.70):

$$\lambda_R \approx \frac{(15\text{m/s})}{(0.0001\text{s}^{-1})}\cdot\frac{11\text{km}}{1.327\text{km}} = \mathbf{1243\ km}$$

We need to check to ensure that $[C_D \cdot N_{BV}/f_c] < 1$.

[0.004·(0.0113s^{-1})/(0.0001s^{-1})] = 0.452 < 1.

Thus, we can expect our approximate solution should work for this case.

Apply eq. (10.71): W_{ABL} =

$$0.004\cdot\frac{(1.327\text{km})}{(11\text{km})}\cdot(1.243\times10^6\text{m})\cdot(6\times10^{-5}\text{s}^{-1})$$
$$\cdot\left[1-0.5\cdot(0.004)\cdot\frac{1243\text{km}}{11\text{km}}\right]$$

= (0.036 m s^{-1}) · [0.774] = **0.028 m s^{-1}**

Check: Physics and units are reasonable.

Exposition: The updraft speed 2.8 cm s^{-1} is slow, but over many hours can cause significant lifting. As the rising air cools adiabatically, clouds form and latent heat is released due to condensation. Hence, clouds and bad weather are often associated with midlatitude cyclones.

Eq. (10.66) can be modified to use geostrophic vorticity. The resulting Ekman pumping at the atmospheric boundary layer top in a midlatitude cyclone is:

$$W_{ABL} = C_D\cdot\frac{G}{f_c}\cdot\zeta_g\cdot\left[1-0.5\frac{C_D\cdot N_{BV}}{f_c}\right] \quad (10.69)$$

The first four factors on the right side imply that larger drag coefficients (i.e., rougher terrain with more trees or buildings) and stronger pressure gradients (as indicated by larger geostrophic wind) driving winds around smaller radii of curvature (i.e., larger geostrophic vorticity) at lower latitudes (i.e., smaller f_c) create stronger updrafts. Also, stronger static stabilities (i.e., larger Brunt-Väisälä frequency NBV) in the troposphere above atmospheric boundary-layer top reduce updraft speed by opposing vertical motion.

One can write an **internal Rossby deformation radius** based on the eq. (10.67) approximation for depth z_i of the atmospheric boundary layer:

$$\lambda_R \approx \frac{G}{f_c}\cdot\frac{z_T}{z_i} \quad (10.70)$$

where tropospheric depth is z_T. Internal and external Rossby deformation radii are described further in the General Circulation and Fronts & Airmasses chapters, respectively.

The Rossby deformation radius can be used to write yet another expression for Ekman pumping vertical velocity out of the top of the atmospheric boundary layer:

$$W_{ABL} = C_D\cdot\frac{z_i}{z_T}\cdot\lambda_R\cdot\zeta_g\cdot\left[1-0.5\cdot C_D\cdot\frac{\lambda_R}{z_T}\right] \quad (10.71)$$

10.9. KINEMATICS

Kinematics is the study of patterns of motion, without regard to the forces that cause them. We will focus on horizontal divergence, vorticity, and deformation. All have units of s^{-1}.

We have already encountered horizontal **divergence**, D, the spreading of air:

$$D = \frac{\Delta U}{\Delta x} + \frac{\Delta V}{\Delta y} \quad (10.72)$$

Figure 10.29a shows an example of pure divergence. Its sign is positive for divergence, and negative for **convergence** (when the wind arrows point toward a common point).

Vorticity describes the rotation of air (Fig. 10.29b). The relative vorticity, ζ_r, about a locally vertical axis is given by:

$$\zeta_r = \frac{\Delta V}{\Delta x} - \frac{\Delta U}{\Delta y} \qquad (10.73)$$

The sign is positive for counterclockwise rotation (i.e., cyclonic rotation in the N. Hemisphere), and negative for clockwise rotation. Vorticity is discussed in greater detail in the General Circulation chapter. Neither divergence nor vorticity vary with rotation of the axes — they are **rotationally invariant**.

Two types of **deformation** are stretching deformation and shearing deformation (Figs. 10.29c & d). **Stretching deformation**, F_1, is given by:

$$F_1 = \frac{\Delta U}{\Delta x} - \frac{\Delta V}{\Delta y} \qquad (10.74)$$

The axis along which air is being stretched (Fig. 10.29c) is called the **axis of dilation** (x axis in this example), while the axis along which air is compressed is called the **axis of contraction** (y axis in this example).

Shearing deformation, F_2, is given by:

$$F_2 = \frac{\Delta V}{\Delta x} + \frac{\Delta U}{\Delta y} \qquad (10.75)$$

As you can see in Fig. 10.29d, shearing deformation is just a rotated version of stretching deformation. The **total deformation**, F, is:

$$F = \left[F_1{}^2 + F_2{}^2 \right]^{1/2} \qquad (10.76)$$

Deformation often occurs along fronts. Most real flows exhibit combinations of divergence, vorticity, and deformation.

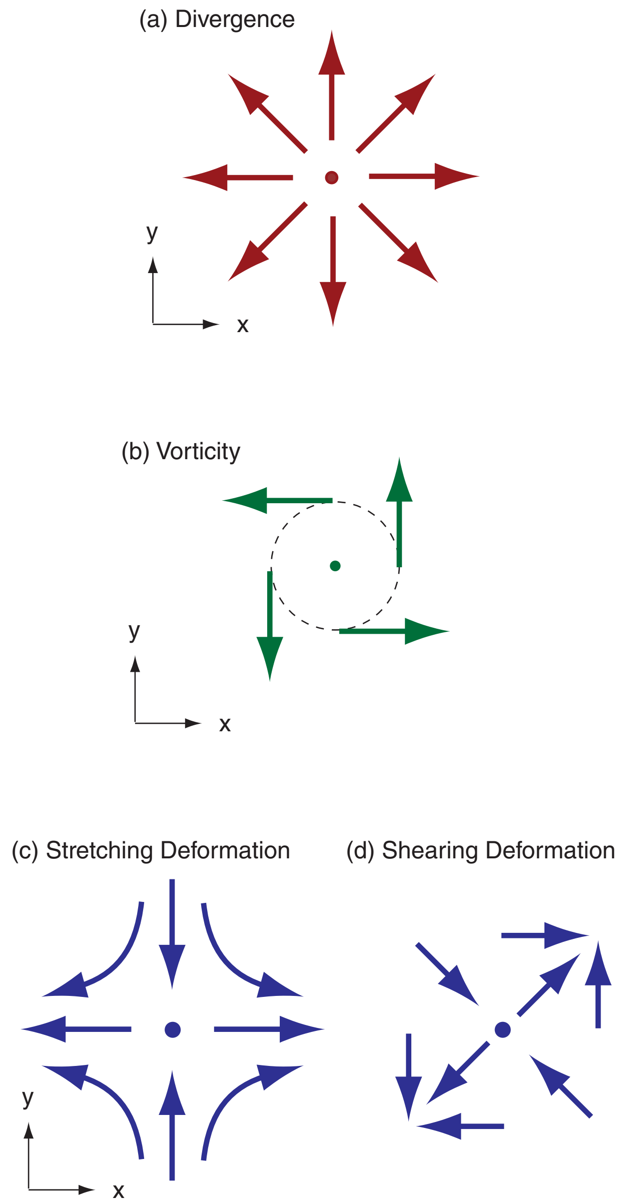


Figure 10.29
Kinematic flow-field definitions. Black arrows represent wind velocity.

10.10. MEASURING WINDS

For weather stations at the Earth's surface, wind direction can be measured with a **wind vane** mounted on a vertical axle. **Fixed vanes** and other shapes can be used to measure wind speed, by using strain gauges to measure the minute deformations of the object when the wind hits it.

The generic name for a wind-speed measuring device is an **anemometer**. A **cup anemometer** has conic- or hemispheric-shaped cups mounted on spokes that rotate about a vertical axle. A **pro-**

peller anemometer has a propeller mounted on a horizontal axle that is attached to a wind vane so it always points into the wind. For these anemometers, the rotation speed of the axle can be calibrated as a wind speed.

Other ways to measure wind speed include a **hot-wire** or **hot-film anemometer**, where a fine metal wire is heated electrically, and the power needed to maintain the hot temperature against the cooling effect of the wind is a measure of wind speed. A **pitot tube** that points into the wind measures the dynamic pressure as the moving air stagnates in a dead-end tube. By comparing this dynamic pressure with the static pressure measured by a different sensor, the pressure difference can be related to wind speed.

Sonic anemometers send pulses of sound back and forth across a short open path between two opposing transmitters and receivers (transceivers) of sound. The speed of sound depends on both temperature and wind speed, so this sensor can measure both by comparing sound travel times in opposite directions. Tracers such as smoke, humidity fluctuations, or clouds can be **tracked** photogrammetrically from the ground or from remote sensors such as laser radars (**lidars**) or satellites, and the wind speed then estimated from the change of position of the tracer between successive images.

Measurements of wind vs. height can be made with **rawinsonde balloons** (using a GPS receiver in the sonde payload to track horizontal drift of the balloons with time), **dropsondes** (like rawinsondes, only descending by parachute after being dropped from aircraft), **pilot balloons** (carrying no payload, but being tracked instead from the ground using radar or theodolites), **wind profilers**, **Doppler weather radar** (see the Satellites & Radar chapter), and via anemometers mounted on aircraft.

10.11. REVIEW

According to Newton's second law, winds are driven by forces. The pressure-gradient creates a force, even in initially calm (windless) conditions. This force points from high to low pressure on a constant altitude chart (such as at sea-level), or points from high to low heights on an isobaric chart (such as the 50 kPa chart). Pressure-gradient force is the main force that drives the winds.

Other forces exist only when there is already a wind. One example is turbulent drag against the ground, which pushes opposite to the atmospheric boundary-layer wind direction. Another example is Coriolis force, which is related to centrifugal force of winds relative to a rotating Earth.

If all the forces vector-sum to zero, then there is no net force and winds blow at constant speed. Theoretical winds based on only a small number of forces are given special names. The geostrophic wind occurs when pressure-gradient and Coriolis forces balance, causing a wind that blows parallel to straight isobars. For curved isobars around lows and highs, the imbalance between these two forces turns the wind in a circle, with the result called the gradient wind. Similar winds can exist in the atmospheric boundary layer, where turbulent drag of the air against the Earth's surface slows the wind and causes it to turn slightly to cross the isobars toward low pressure.

Waterspouts and tornadoes can have such strong winds that pressure-gradient force is balanced by centrifugal force, with the resulting wind speed known as the cyclostrophic wind. In oceans, currents can inertially flow in a circle.

The two most important force balances at mid-latitudes are hydrostatic balance in the vertical, and geostrophic balance in the horizontal.

Conservation of air mass gives the continuity equation, for which an incompressible approximation can be used in most places except in thunderstorms. Mechanisms that cause motion in one direction (horizontal or vertical) will also indirectly cause motions in the other direction as the air tries to maintain continuity, resulting in a circulation.

Kinematics is the word that describes the behavior and effect of winds (such as given by the continuity equation) without regard to the forces that cause them. The word **dynamics** describes how forces cause winds (as given by Newton's 2^{nd} law).

10.12. HOMEWORK EXERCISES

Some of these questions are inspired by exercises in Stull, 2000: *Meteorology for Sci. & Engr. 2nd Ed.*, Brooks/Cole, 528 pp.

10.12.1. Broaden Knowledge & Comprehension

For all the exercises in this section, collect information off the internet. Don't forget to cite the web sites you use.

B1. a. Find a weather map showing today's sea-level pressure isobars near your location. Calculate pressure-gradient force (N) based on your latitude and the isobar spacing (km/kPa).

b. Repeat this for a few days, and plot the pressure gradient vs. time.

B2. Get 50 kPa height contour maps (i.e., 500 hPa heights) over any portion of the Northern Hemisphere. In 2 locations at different latitudes having straight isobars, compute the geostrophic wind speed. In 2 locations of curved isobars, compute the gradient wind speed. How do these theoretical winds compare with wind observations near the same locations?

B3. Similar exercise B2, but for 2 locations in the Southern Hemisphere.

B4. a. Using your results from exercise B2 or B3, plot the geostrophic wind speed vs. latitude and pressure gradient on a copy of Fig. 10.10. Discuss the agreement or disagreement of your results vs. the lines plotted in that figure.

b. Using your results from exercise B4 or B5, show that gradient winds are indeed faster than geostrophic around high-pressure centers, and slower around low-pressure centers.

B5. Discuss surprising insights regarding Isaac Newton's discoveries on forces and motion.

B6. Get a map of sea-level pressure, including isobar lines, for a location or date where there are strong low and high-pressure centers adjacent to each other. On a printed copy of this map, use a straight edge to draw a line connecting the low and high centers, and extend the line further beyond each center. Arbitrarily define the high center as location $x = 0$. Then, along your straight line, add distance tic marks appropriate for the map scale you are using. For isobars crossing your line, create a table that lists each pressure P and its distance x from the high. Then plot P vs. x and discuss how it compares with Fig. 10.14. Discuss the shape of your curve in the low- and high-pressure regions.

B7. Which animations best illustrate Coriolis Force?

B8. a. Get a map of sea-level pressure isobars that also shows observed wind directions. Discuss why the observed winds have a direction that crosses the isobars, and calculate a typical crossing angle..

b. For regions where those isobars curve around cyclones or anticyclones, confirm that winds spiral into lows and out of highs.

c. For air spiraling in toward a cyclone, estimate the average inflow radial velocity component, and calculate W_{BL} based on incompressible continuity.

B9. For a typhoon or hurricane, get a current or past weather map showing height-contours for any one isobaric level corresponding to an altitude about 1/3 the altitude of the storm (i.e., a map for any pressure level between 85 to 60 kPa). At the eye-wall location, use the height-gradient to calculate the cyclostrophic wind speed. Compare this with the observed hurricane winds at that same approximate location, and discuss any differences.

B10. Get a 500 hPa (= 50 kPa) geopotential height contour map that is near or over the equator. Compute the theoretical geostrophic wind speed based on the height gradients at 2 locations on that map where there are also observed upper-air wind speeds. Explain why these theoretical wind speeds disagree with observed winds.

10.12.2. Apply

A1. Plot the wind symbol for winds with the following directions and speeds:

a. N at 5 kt b. NE at 35 kt c. E at 65 kt
d. SE at 12 kt e. S at 48 kt f. SW at 105 kt
g. W at 27 kt h. NW at 50 kt i. N at 125 kt

A2. How fast does an 80 kg person accelerate when pulled with the force given below in Newtons?

a. 1 b. 2 c. 5 d. 10 e. 20 f. 50
g. 100 h. 200 i. 500 j. 1000 k. 2000

A3. Suppose the following force per mass is applied on an object. Find its speed 2 minutes after starting from rest.

a. 5 N kg^{-1} b. 10 m·s^{-2}
c. 15 N kg^{-1} d. 20 m·s^{-2}
e. 25 N kg^{-1} f. 30 m·s^{-2}
g. 35 N kg^{-1} h. 40 m·s^{-2}
i. 45 N kg^{-1} j. 50 m·s^{-2}

A4. Find the advective "force" per unit mass given the following wind components (m s^{-1}) and horizontal distances (km):

a. U=10, ΔU=5, Δx=3
b. U=6, ΔU=–10, Δx=5
c. U=–8, ΔV=20, Δx=10
d. U=–4, ΔV=10, Δx=–2
e. V=3, ΔU=10, Δy=10
f. V=–5, ΔU=10, Δy=4
g. V=7, ΔV=–2, Δy=–50
h. V=–9, ΔV=–10, Δy=–6

A5. Town A is 500 km west of town B. The pressure at town A is given below, and the pressure at town B is 100.1 kPa. Calculate the pressure-gradient force/mass in between these two towns.

a. 98.6 b. 98.8 c. 99.0 d. 99.2 e. 99.4

f. 99.6	g. 99.8	h. 100.0	i. 100.2	j. 100.4
k. 100.6	l. 100.8	m. 101.0	n. 101.2	o. 101.4

A6. Suppose that $U = 8$ m s^{-1} and $V = -3$ m s^{-1}, and latitude = 45° Calculate centrifugal-force components around a:
a. 500 km radius low in the N. hemisphere
b. 900 km radius high in the N. hemisphere
c. 400 km radius low in the S. hemisphere
d. 500 km radius high in the S. hemisphere

A7. What is the value of f_c (Coriolis parameter) at:
a. Shanghai
b. Istanbul
c. Karachi
d. Mumbai
e. Moscow
f. Beijing
g. São Paulo
h. Tianjin
i. Guangzhou
j. Delhi
k. Seoul
l. Shenzhen
m. Jakarta
n. Tokyo
o. Mexico City
p. Kinshasa
q. Bangalore
r. New York City
s. Tehran
t. (a city specified by your instructor)

A8. What is the magnitude and direction of Coriolis force/mass in Los Angeles, USA, given:

	U (m s^{-1})	V (m s^{-1})
a.	5	0
b.	5	5
c.	5	–5
d.	0	5
e.	0	–5
f.	–5	0
g.	–5	–5
h.	–5	5

A9. Same wind components as exercise A8, but find the magnitude and direction of turbulent drag force/mass in a statically neutral atmospheric boundary layer over an extensive forested region.

A10. Same wind components as exercise A8, but find the magnitude and direction of turbulent drag force/mass in a statically unstable atmospheric boundary layer with a 50 m/s buoyant velocity scale.

A11. Draw a northwest wind of 5 m s^{-1} in the S. Hemisphere on a graph, and show the directions of forces acting on it. Assume it is in the boundary layer.
a. pressure gradient b. Coriolis
c. centrifugal d. drag

A12. Given the pressure gradient magnitude (kPa/1000 km) below, find geostrophic wind speed for a location having $f_c = 1.1\text{x}10^{-4}$ s^{-1} and $\rho = 0.8$ kg m^{-3}.

a. 1	b. 2	c. 3	d. 4	e. 5
f. 6	g. 7	h. 8	i. 9	j. 10
k. 11	m. 12	n. 13	o. 14	p. 15

A13. Suppose the height gradient on an isobaric surface is given below in units of (m km^{-1}). Calculate the geostrophic wind at 55°N latitude.

a. 0.1	b. 0.2	c. 0.3	d. 0.4	e. 0.5
f. 0.6	g. 0.7	h. 0.8	i. 0.9	j. 1.0
k. 1.1	m. 1.2	n. 1.3	o. 1.4	p. 1.5

A14. At the radius (km) given below from a low-pressure center, find the gradient wind speed given a geostrophic wind of 8 m s^{-1} and given $f_c = 1.1\text{x}10^{-4}$ s^{-1}.

a. 500	b. 600	c. 700	d. 800	e. 900
f. 1000	g. 1200	h. 1500	i. 2000	j. 2500

A15. Suppose the geostrophic winds are $U_g = -3$ m s^{-1} with $V_g = 8$ m s^{-1} for a statically-neutral boundary layer of depth $z_i = 1500$ m, where $f_c = 1.1\text{x}10^{-4}$ s^{-1}. For drag coefficients given below, what is the atmos. boundary-layer wind speed, and at what angle does this wind cross the geostrophic wind vector?

a. 0.002	b. 0.004	c. 0.006	d. 0.008	e. 0.010
f. 0.012	g. 0.014	h. 0.016	i. 0.018	j. 0.019

A16. For a statically unstable atmos. boundary layer with other characteristics similar to those in exercise A15, what is the atmos. boundary-layer wind speed, at what angle does this wind cross the geostrophic wind vector, given w_B (m s^{-1}) below?

a. 75	b. 100	c. 50	d. 200	e. 150
f. 225	g. 125	h. 250	i. 175	j. 275

A17(§). Review the Sample Application in the "Atmospheric Boundary Layer Gradient Wind" section. Re-do that calculation for M_{ABLG} with a different parameter as given below:
a. $z_i = 1$ km b. $C_D = 0.003$ c. $G = 8$ m s^{-1}
d. $f_c = 1.2\text{x}10^{-4}$ s^{-1} e. $R = 2000$ km
f. $G = 15$ m s^{-1} g. $z_i = 1.5$ km h. $C_D = 0.005$
i. $R = 1500$ km j. $f_c = 1.5\text{x}10^{-4}$ s^{-1}
Hint: Assume all other parameters are unchanged.

A18. Find the cyclostrophic wind at radius (m) given below, for a radial pressure gradient = 0.5 kPa m^{-1}:
a. 10 b. 12 c. 14 d. 16 e. 18
f. 20 g. 22 h. 24 i. 26 j. 28 k 30

A19. For an inertial wind, find the radius of curvature (km) and the time period (h) needed to complete one circuit, given $f_c = 10^{-4}$ s^{-1} and an initial wind speed (m s^{-1}) of:
a. 1 b. 2 c. 3 d. 4 e. 6 f. 7 g. 8 h. 9
i. 10 j. 11 k. 12 m. 13 n. 14 o. 15

A20. Find the antitriptic wind for the conditions of exercise A15.

A21. Below is given an average inward radial wind component (m s^{-1}) in the atm. boundary layer at radius 300 km from the center of a cyclone. What is the average updraft speed out of the atm. boundary-layer top, for a boundary layer that is 1.2 km thick?
a. 2 b. 1.5 c. 1.2 d. 1.0
e. –0.5 f. –1 g. –2.5 h. 3 i. 0.8 j. 0.2

A22. Above an atmospheric boundary layer, assume the tropospheric temperature profile is $\Delta T/\Delta z = 0$. For a midlatitude cyclone, estimate the atm. boundary-layer thickness given a near-surface geostrophic wind speed (m s^{-1}) of:
a. 5 b. 10 c. 15 d. 20 e. 25 f. 30
g. 35 h. 40 i. 3 j. 8 k. 2 l. 1

A23(§). For atm. boundary-layer pumping, plot a graph of updraft velocity vs. geostrophic wind speed assuming an atm. boundary layer of depth 0.8 km, a drag coefficient 0.005 . Do this only for wind speeds within the valid range for the atmos. boundary-layer pumping eq. Given a standard atmospheric lapse rate at 30° latitude with radius of curvature (km) of:
a. 750 b. 1500 c. 2500 d. 3500 e. 4500
f. 900 g. 1200 h. 2000 i. 3750 j. 5000

A24. At 55°N, suppose the troposphere is 10 km thick, and has a 10 m s^{-1} geostrophic wind speed. Find the internal Rossby deformation radius for an atmospheric boundary layer of thickness (km):
a. 0.2 b. 0.4 c. 0.6 d. 0.8 e. 1.0
f. 1.2 g. 1.5 h. 1.75 i. 2.0 j. 2.5

A25. Given $\Delta U/\Delta x = \Delta V/\Delta x$ = (5 m s^{-1}) / (500 km), find the divergence, vorticity, and total deformation for ($\Delta U/\Delta y$, $\Delta V/\Delta y$) in units of (m s^{-1})/(500 km) as given below:
a. (–5, –5) b. (–5, 0) c. (0, –5) d. (0, 0) e. (0, 5)
f. (5, 0) g. (5, 5) h. (–5, 5) i. (5, –5)

10.12.3. Evaluate & Analyze

E1. Discuss the relationship between eqs. (1.24) and (10.1).

E2. Suppose that the initial winds are unknown. Can a forecast still be made using eqs. (10.6)? Explain your reasoning.

E3. Considering eq. (10.7), suppose there are no forces acting. Based on eq. (10.5), what can you anticipate about the wind speed.

E4. We know that winds can advect temperature and humidity, but how does it work when winds advect winds? Hint, consider eqs. (10.8).

E5. For an Eulerian system, advection describes the influence of air that is blown into a fixed volume. If that is true, then explain why the advection terms in eq. (10.8) is a function of the wind gradient (e.g., $\Delta U/\Delta x$) instead of just the upwind value?

E6. Isobar packing refers to how close the isobars are, when plotted on a weather map such as Fig. 10.5. Explain why such packing is proportional to the pressure gradient.

E7. Pressure gradient has a direction. It points toward low pressure for the Northern Hemisphere. For the Southern Hemisphere, does it point toward high pressure? Why?

E8. To help you interpret Fig. 10.5, consider each horizontal component of the pressure gradient. For an arbitrary direction of isobars, use eqs. (10.9) to demonstrate that the vector sum of the components of pressure-gradient do indeed point away from high pressure, and that the net direction is perpendicular to the direction of the isobars.

E9. For centrifugal force, combine eqs. (10.13) to show that the net force points outward, perpendicular to the direction of the curved flow. Also show that the magnitude of that net vector is a function of tangential velocity squared.

E10. Why does $f_c = 0$ at the equator for an air parcel that is stationary with respect to the Earth's surface, even though that air parcel has a large tangential velocity associated with the rotation of the Earth?

E11. Verify that the net Coriolis force is perpendicular to the wind direction (and to its right in the N. Hemisphere), given the individual components described by eqs. (10.17).

E12. For the subset of eqs. (10.1 - 10.17) defined by your instructor, rewrite them for flow in the Southern Hemisphere.

E13. Verify that the net drag force opposes the wind by utilizing the drag components of eqs. (10.19). Also, confirm that drag-force magnitude for statically neutral conditions is a function of wind-speed squared.

E14. How does the magnitude of the turbulent-transport velocity vary with static stability, such as between statically unstable (convective) and statically neutral (windy) situations?

E15. Show how the geostrophic wind components can be combined to relate geostrophic wind speed to pressure-gradient magnitude, and to relate geostrophic wind direction to pressure-gradient direction.

E16. How would eqs. (10.26) for geostrophic wind be different in the Southern Hemisphere?

E17. Using eqs. (10.26) as a starting point, show your derivation for eqs. (10.29).

E18. Why are actual winds finite near the equator even though the geostrophic wind is infinite there? (Hint, consider Fig. 10.10).

E19. Plug eq. (10.33) back into eqs. (10.31) to confirm that the solution is valid.

E20. Plug eqs. (10.34) back into eq. (10.33) to confirm that the solution is valid.

E21. Given the pressure variation shown in Fig. 10.14. Create a mean-sea-level pressure weather map with isobars around high- and low-pressure centers such that the isobar packing matches the pressure gradient in that figure.

E22. Fig. 10.14 suggests that any pressure gradient is theoretically possible adjacent to a low-pressure center, from which we can further infer that any wind speed is theoretically possible. For the real atmosphere, what might limit the pressure gradient and the wind speed around a low-pressure center?

E23. Given the geopotential heights in Fig. 10.3, calculate the theoretical values for gradient and/or geostrophic wind at a few locations. How do the actual winds compare with these theoretical values?

E24. Eq. 10.39 is an "implicit" solution. Why do we say it is "implicit"?

E25. Determine the accuracy of explicit eqs. (10.41) by comparing their approximate solutions for ABL wind against the more exact iterative solutions to the implicit form in eq. (10.39).

E26. No explicit solution exists for the neutral atmospheric boundary layer winds, but one exists for the statically unstable ABL? Why is that?

E27. Plug eqs. (10.42) into eqs (10.38) or (10.39) to confirm that the solution is valid.

E28(§). a. Create your own spreadsheet that gives the same answer for ABLG winds as in the Sample Application in the ABLG-wind section.

b. Do "what if" experiments with your spreadsheet to show that the full equation can give the gradient wind, geostrophic wind, and boundary-layer wind for conditions that are valid for those situations.

c. Compare the results from (b) against the respective analytical solutions (which you must compute yourself).

E29. Photocopy Fig. 10.13, and enhance the copy by drawing additional vectors for the atmospheric boundary-layer wind and the ABLG wind. Make these vectors be the appropriate length and direction relative to the geostrophic and gradient winds that are already plotted.

E30. Plug the cyclostrophic-wind equation into eq. (10.45) to confirm that the solution is valid for its special case.

E31. Find an equation for cyclostrophic wind based on heights on an isobaric surface. [Hint: Consider eqs. (10.26) and (10.29).]

E32. What aspects of the *Approach to Geostrophy* INFO Box are relevant to the inertial wind? Discuss.

E33. a. Do your own derivation for eq. (10.66) based on geometry and mass continuity (total inflow = total outflow).

b. Drag normally slows winds. Then why does the updraft velocity increase in eq. (10.66) as drag coefficient increases?

c. Factor b varies negatively with increasing drag coefficient in eq. (10.66). Based on this, would you change your argument for part (b) above?

E34. Look at each term within eq. (10.69) to justify the physical interpretations presented after that equation.

E35. Consider eq. (10.70). For the internal Rossby deformation radius, discuss its physical interpretation in light of eq. (10.71).

E36. What type of wind would be possible if the only forces were turbulent-drag and Coriolis. Discuss.

E37. Derive equations for Ekman pumping around anticyclones. Physically interpret your resulting equations.

E38. Rewrite the total deformation as a function of divergence and vorticity. Discuss.

10.12.4. Synthesize

S1. For zonal (east-west) winds, there is also a vertical component of Coriolis force. Using your own diagrams similar to those in the INFO box on Coriolis Force, show why it can form. Estimate its magnitude, and compare the magnitude of this force to other typical forces in the vertical. Show why a vertical component of Coriolis force does not exist for meridional (north-south) winds.

S2. On Planet Cockeyed, turbulent drag acts at right angles to the wind direction. Would there be anything different about winds near lows and highs on Cockeyed compared to Earth?

S3. The time duration of many weather phenomena are related to their spatial scales, as shown by eq. (10.53) and Fig. 10.24. Why do most weather phenomena lie near the same diagonal line on a log-log plot? Why are there not additional phenomena that fill out the relatively empty upper and lower triangles in the figure? Can the distribution of time and space scales in Fig. 10.24 be used to some advantage?

S4. What if atmospheric boundary-layer drag were constant (i.e., not a function of wind speed). Describe the resulting climate and weather.

S5. Suppose Coriolis force didn't exist. Describe the resulting climate and weather.

S6. Incompressibility seems like an extreme simplification, yet it works fairly well? Why? Consider what happens in the atmosphere in response to small changes in density.

S7. The real Earth has locations where Coriolis force is zero. Where are those locations, and what does the wind do there?

S8. Suppose that wind speed $M = c{\cdot}F/m$, where c = a constant, m = mass, and F is force. Describe the resulting climate and weather.

S9. What if Earth's axis of rotation was pointing directly to the sun. Describe the resulting climate and weather.

S10. What if there was no limit to the strength of pressure gradients in highs. Describe the resulting climate, winds and weather.

S11. What if both the ground and the tropopause were rigid surfaces against which winds experience turbulent drag. Describe the resulting climate and weather.

S12. If the Earth rotated half as fast as it currently does, describe the resulting climate and weather.

S13. If the Earth had no rotation about its axis, describe the resulting climate and weather.

S14. Consider the Coriolis-force INFO box. Create an equation for Coriolis-force magnitude for winds that move:
a. westward b. southward

S15. What if a cyclostrophic-like wind also felt drag near the ground? This describes conditions at the bottom of tornadoes. Write the equations of motion for this situation, and solve them for the tangential and radial wind components. Check that your results are reasonable compared with the pure cyclostrophic winds. How would the resulting winds affect the total circulation in a tornado? As discoverer of these winds, name them after yourself.

S16. What if $F = c{\cdot}a$, where c = a constant not equal to mass, a = acceleration, and F is force. Describe the resulting dynamics of objects such as air parcels.

S17. What if pressure-gradient force acted parallel to isobars. Would there be anything different about our climate, winds, and weather maps?

S18. For a free-slip Earth surface (no drag), describe the resulting climate and weather.

S19. Anders Persson discussed issues related to Coriolis force and how we understand it (see *Weather*, 2000.) Based on your interpretation of his paper, can Coriolis force alter kinetic energy and momentum of air parcels, even though it is only an apparent force? Hint, consider whether Newton's laws would be violated if your view these motions and forces from a fixed (non-rotating) framework.

S20. If the Earth was a flat disk spinning about the same axis as our real Earth, describe the resulting climate and weather.

S21. Wind shear often creates turbulence, and turbulence mixes air, thereby reducing wind shear. Considering the shear at the ABL top in Fig. 10.7, why can it exist without mixing itself out?

S22. Suppose there was not centrifugal or centripetal force for winds blowing around lows or highs. Describe the resulting climate, winds and weather.

S23. Suppose advection of the wind by the wind were impossible. Describe the resulting climate and weather.